PHYSICS I EXPERIMENTS LABORATORY BOOK

For Engineering Students

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ESKİŞEHİR 2026

Eskişehir Osmangazi Üniversitesi Yayınları No: 412

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ISBN NO
605-9975-98-8

Baskı Tesisleri
ESOGÜ Basımevi

T.C. Kültür ve Turizm Bakanlığı Matbaa Sertifika No.: 64281

ESKİŞEHİR 2026

STUDENT'S

NAME and SURNAME :...

DEPARTMENT : ....

NUMBER : ....

No	Exp. Code	Experiment Name	Date	Teaching Staff Sign
1	M	Measurement, Error Calculation and Graph Plotting In Physics		
2	M1	Motion with Constant Acceleratıon and Conservation Of Linear Momentum		
3	M2	Ballistic Pendulum and Projectile Motion		
4	M3	Free Fall and Atwood's Machine		
5	M4	Simple Pendulum and Conservation Of Energy		
6	M5	Motion On a Frictional Inclined Surface		
7	M6	Springs, Pulleys and Pulley Blocks		
8	M7	Rotational Motion		
9	M8	Centripetal Force		
10	M9	Physical Pendulum and Moment of Inertia		
11	M10	Viscosity		
12	M11	Archimedes' Principle and Density		

PREFACE

Physics is a natural science that deals with the study of phenomena, how these occur in nature and structure and behavior of matter. Physics as a natural science sets ground for other natural, engineering, and technological science branches. Physics is based on experimental observations and quantitative measurements. Physical quantities, how to measure and what physics law describes these quantities are necessary in order to describe the meaningful relations to explain the nature. In this context, laboratory studies are of particular importance in Physics.

This Physics I Laboratory book contains mechanics experiments in Physics I Laboratory course of fall semester for Eskişehir Osmangazi University Faculty of Engineering and Architecture students. Our goal to write this book is to teach these students fundamental principles and concepts of Physics by means of observations and to help them apply their knowledge into engineering subjects. Physics I Laboratory book will contribute to students of this course to comprehend the subjects of mechanics and develop commentary skills in comparing theoretical outcomes and experimental results within the limits of experimental errors. In the context of Physics I Laboratory course, the following experiments will be performed: Measurement, error calculation and graph plotting in Physics; motion at constant acceleration and conservation of linear momentum; ballistic pendulum and projectile motion; free fall and Atwood's machine; simple pendulum and conservation of energy; motion on a frictional inclined plane; springs, pulleys, and pulley blocks; rotational motion; centripetal force; physical pendulum and moment of inertia; viscosity, Archimedes' Principle and density. The goal of each experiment is different from each other, however common point of all experiments is to observe, measure, and calculate related quantities, to learn and find quantities from graph drawing, to determine uncertainties in measurements, and to make comments on experimental results. The goal, theoretical background, and experimental procedure of each experiment in Physics I Laboratory Book are given explicitly for students to evaluate experimental results. We wish this book to be helpful to students of Faculty of Engineering and Architecture in applying their knowledge to their professional areas. Furthermore, support, encouragement, and critics of teaching staff and students are important for us to improve this book. Therefore, we remark that any suggestion to contribute to this text is very welcome in this sense.

We thank to the staff members of Physics Department in the preparation of this book, Assoc. Prof. Dr. Sertaç EROĞLU and Research Assis. Dr. Murat KELLEGÖZ who contributed to the earlier version of Physics I Laboratory book, and Halil Yasin ADIYAMAN who helped to set the experiments for their usage actively.

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PHYSICS LABORATORY
WORKING INSTRUCTIONS

1. Students should come to the lab on time. **Students who arrive later than 10 minutes are not allowed to enter the laboratory.**
2. Experimental groups and loop scheme are announced on the laboratory board. **Students cannot change their groups arbitrarily.**
3. **Students should obey laboratory staff's cautions.** Laboratory staff have the authority to make any changes to keep the course layout.
4. **Avoid disrupting the course by walking around the tables or making noise in the lab.**
5. **Keep your cell phones switched off during the course.** It is not enough that your phone is on silent mode.
6. Students should bring their necessary tools with themselves, **such as pen, pencil, pencil leads, eraser, ruler, calculator, etc.**
7. Before coming to the laboratory, students should be prepared for the theory and experimental procedure using **Physics Laboratory Book.**
8. **Students should also be prepared for pre- and post-oral examination by laboratory staff.**
9. Before the experiment, students should check experimental equipment and **inform laboratory staff for any damage and loss.** In the case of any experimental damage, responsibility belongs to the members of the relevant experiment.
10. **Students should not set or start the experiment before staff's approval.**
11. The students who take the laboratory course prepares a report for every experiment they perform. "The experimental grade" will be calculated by taking the average of these report scores.
12. Report grading consists of the use of experimental instruments, your performance, discussion of the experiment, and how tidy you leave your experimental equipment. Detailed grading is done with report layout, results, graph drawing, evaluation, and success during the experiment.
13. There will not be any laboratory course during the midterm weeks.
14. At the end of each experiment, when submitting in your report, sign the attendance list. Note that you cannot claim any right for your reports that you threw under the staff's door without an approval signature.

15. **The case of student's nonattendance to more than 3 (three) experiments results in failure of the course as an absence.** Students are allowed to make up their one or two experiments during the make-up week if they are excused absent with a medical report or official permission. The grades for uncompleted experiment(s) will be considered as **0 (zero).**
16. Make-up week and students will be announced after the completion of the experiments.
17. At the end of each experiment, students should leave experimental equipment and their table clean and tidy.

GOOD LUCK…

CONTENTS

MEASUREMENT, ERROR CALCULATION, AND GRAPH PLOTING IN PHYSICS

M

MEASUREMENT, ERROR CALCULATION, AND GRAPH PLOTING IN PHYSICS

M

PURPOSE OF THE EXPERIMENT:

1. To learn how to use of a compass and a micrometer.
2. To measure dimensions of a shapely object.
3. To measure a mass of a shapely object by using a balance scale and digital scale.
4. To calculate the density of a shapely object of a known mass.
5. To learn error calculation and graph plotting.

EQUIPMENT FOR THE EXPERIMENT:

Set of objects with different geometry

Digital scale

Balance scale

Set of masses

Micrometer

Compass

Ruler

THEORY:

PHYSICS AND MEASUREMENT

Humanity has been interested in his environmental circumstances since the very early times. Physics covers investigation of environmental circumstances, how they exist, and which rules apply. Physics also includes the study of behavior and structure of matter.

Physical science is based on experimental observations and quantitative measurements. It is important to know physical quantities and how to measure them in order to explain the nature. Each one of the words such as l*ength, time, mass, speed, velocity, acceleration, force, momentum, electric charge, potential, etc.* corresponds to a physical quantity and we use many of them in our daily life. Sometimes, they have different and important meanings in Physics. Several experiments have been carried out in Physics Laboratories. Even though these experiments may have different purposes, they have common targets, i.e., measurement, recording the quantities, determining indirectly other quantities from measured and recorded quantities. For example, the distance and traveling time between two points can be measured in order to calculate speed and velocity. Another example is that a wire with resistance can be connected to a battery, then the current on the wire and potential can be measured in order to calculate wire resistance.

Physical quantities are not totally independent of each other as seen in previous examples. Physics contains seven fundamental quantities: ***'Length', 'mass', and 'time'*** are the first three quantities used in Mechanics. The other quantities used in Mechanics are derived from these three quantities. ***'Electric current', 'thermodynamic temperature', 'amount of substance', and 'light intensity'*** are the other four quantities. Standards have been determined in order to perform precise measurements. This system is called SI unit system and is given Table 1.

TABLE 1. Fundamental quantities, SI units, and symbols.

Quantities	SI Unit	Symbol
Length	Meter	m
Mass	Kilogram	kg
Time	Second	s
Electric current	Amper	A
Thermodynamic temperature	Kelvin	K
Amount of substance	Mole	mol
Light intensity	Candela	cd

Bigger and smaller quantities in metric system are defined by multiplying a standard unit by multiples of 10. Table 2 shows upper and lower factors of 10 in SI units.

TABLE 2. Prefixes used with SI units.

Lower factors of 10	Prefix	Symbol	Upper factors of 10	Prefix	Symbol
10^{-24}	Yocto	y	10^{1}	Deka	da
10^{-21}	Zepto	z	10^{2}	Hecto	h
10^{-18}	Atto	a	10^{3}	Kilo	k
10^{-15}	Femto	f	10^{6}	Mega	M
10^{-12}	Pico	p	10^{9}	Giga	G
10^{-9}	Nano	n	10^{12}	Tera	T
10^{-6}	Micro	µ	10^{15}	Peta	P
10^{-3}	Milli	m	10^{18}	Exa	E
10^{-2}	Centi	c	10^{21}	Zetta	Z
10^{-1}	Deci	d	10^{24}	Yotta	Y

Measurements can be seperated into two groups: '***direct measurements***' and '***indirect measurements***'. Direct measurement of a physical quantity is obtained by reading the scale of a measurement device which is calibrated to correspond to the unit of the quantity. Measurement of a length by a ruler is an example for a direct measurement. Indirect measurement is the measurement of a physical quantity such that their values can not be determined by a measurement device directly. Indirect or calculated measurements are the measurements of physical quantities obtained by using mathematical equations and data obtained from the direct measurements. For example, volume calculation of a sphere can be determined using the diameter of the sphere. The measurement of the diameter is obtained by a direct measurement.

ERRORS

The difference between real and measured values for the magnitude of a physical quantity is called as '***measurement error***'. There are always errors in physically measured quantities. If the errors are determined, the experimental results will make sense. Therefore, the tolerance of any experiment carried out must be specified.

When teoretical studies are considered, there is no error in a measurement in ideal conditions. On the other hand, when real conditions are considered, error types in any experiment may depend on the observer, experimental setup or design of the experiment. Errors in experiments can be explained by an appropriate evaluation on error origin and a possible maximum error in experiment (the minimum value can be measured with the used measurement device). The measured result is stated with the maximum error using the symbol of '±'. For example, if 'x' is the real value of a measurement and 'Δx' is the maximum error, the measured result will be stated as '$x \pm \Delta x$'.

Errors may occur for different type of reasons. The most important factors result from the measurement devices, experimental calculations, and observed quantity in itself. Errors during any measurement are seperated into two groups: ***'Systematic errors'*** and ***'random errors'***.

Systematic Errors

Errors that are fixed and do not change during experimental measurements are called ***'systematic errors'***. Systematic errors usually result from the measurement device, wrong observations, or external factors before and after the experiment that were not considered. It is very important to minimize possible error sources before starting any experiment by a careful experimental setup and good calibration. The errors caused by defective or bad calibrated devices are named ***'calibration errors'***. This error can be minimized by a good calibration of the measurement device. Any experimenter may cause systematic errors by his habit or by some other factors. The systematic errors can be minimized by letting other persons to perform the same experiment. Furthermore, they can be minimized by controlling external factors such as temperature, humidity, and wind.

Random Errors

Random errors affect the result equally as positive or negative because they are coincidental. As contrary to the systematic errors, random errors are caused by precision of measurement devices and unknown and unforeseen changes during the experiment. For example, a sudden change of voltage in the domestic electrical power supply during the measurement of voltage in an electric circuit will cause a random error. The most useful way to minimize random errors is to repeat the experiment several times.

<u>Random errors in position measurements</u>

Let's consider to perform a measurement with a ruler. Error in using a ruler is equal to the smallest measurement interval on that ruler. For example, taking the measurement for the position of a point A is shown by a ruler in Figure 1. The ruler used in the figure is scaled by "mm" and the point A is found at 3.7 cm, but the exact value may be somewhere in between 3.6 cm and 3.8 cm. Because the ruler is scaled by 1 mm steps, the result of the measurement for the position of a point A must be written as 3.7 ± 0.1 cm. Therefore, a ruler scaled by mm intervals will provide more precise results than a ruler scaled by cm intervals.

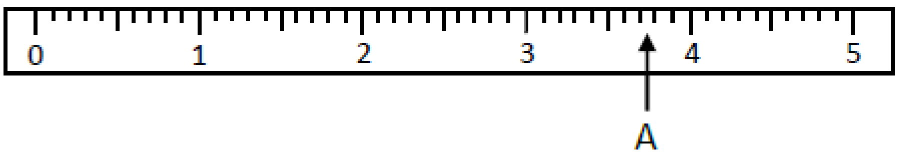


Figure 1. Measurement for the position of a point 'A' by using a ruler.

Random errors in length measurements

Random errors are measurement errors of a distance between two points in one dimension. Let's measure the length of a nail with a ruler as shown in Figure 2. Similar to the masurement of a position, the length measurement is a measurement of two ends 'A' and 'B' on the nail. Therefore, the positions of these two points must be measured with respective to the same reference point. As mentioned before, these position measurements with a ruler are taken with a 1-mm error. Thus, 2-mm error will occur due to the measurements of two points. As shown in Figure 2, the position of the point A wrt point 0 is 1.6 cm and the position of the point B wrt the point 0 is 3.7 cm. By using these data, the length of this pencil is calculated by

$$L \pm \Delta L = (3.7 - 1.6) \pm 0.2 \text{ cm} = 2.1 \pm 0.2 \text{ cm}$$

Effects of random errors can be reduced by improving the experimental conditions, repeating experiments, and taking the average of the data.

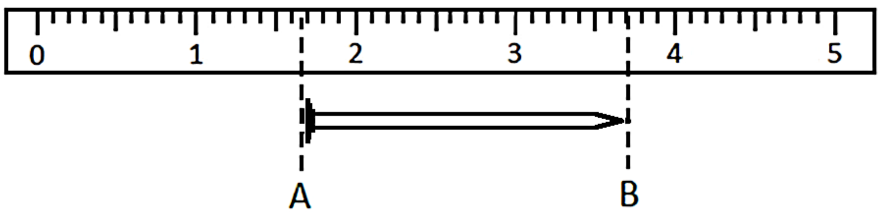


Figure 2. Measurement of a nail by using a ruler.

ACCURACY AND PRECISION OF A MEASUREMENT

Measurements in Physics Laboratories are carried out by using several tools such as ruler, voltmeter, and ammeter. There are always some uncertainties in measurements. Therefore, the main goal of a measurement is to use a proper tool and to be careful during the experiment in order to get the expected or the closest result of the real value. Reliability of a measured quantity is analyzed by '**precision'** and **'accuracy'** of these experimental results. Precision and accuracy are two important factors to consider when data is collected. The closeness of the measured result to the real value is called as ***'precision of the measurement'*** and closeness of experimental data to each other is called as ***'accuracy of the measurement'***.

Accuracy of an experiment generally depends on systematic and random errors. On the other hand, precision of an experiment mostly depends on random errors. We need to minimize measurement errors and provide information about precision and accuracy of the measurement at the end of an experiment because we can never find the real value in an experiment. If an experiment is performed only once, the largest error (absolute error) will be the smallest interval on the measurement tool that is used. The result of the measurement is given by

$$X = X_{measured} \pm \text{(the smallest interval)}$$

$$X = X_{measured} \pm \Delta X.$$

If the measurement is repeated more than once, statistical calculations can be used:

Average Value ($\overline{X}$): Assume that an experiment is repeated *n* times for a quantity that has a real value of X_0 and the measurements leads to $X_1, X_2, X_3, ..., X_n$. The average value of these measurements, $\left(\overline{X}\right)$ is given by

$$\overline{X} = \frac{X_1 + X_2 + X_3 + ... + X_n}{n} = \frac{\sum_{i=1}^{n} X_i}{n} \qquad \left(i = 1, 2, ..., n\right).$$

There will be a difference between each measured value and the average value of all measurements. This difference is named as '***deviation of the measurement'*** or ***'amount of deviation'*** and given by

$$d_i = \left|X_i - \overline{X}\right|.$$

Note that the deviation is given in the absolute value.

Then the average absolute error is given by

$$\Delta X = \sqrt{\frac{d_1^2 + d_2^2 + ... + d_n^2}{n\left(n-1\right)}}$$

Therefore the result is given by

$$X = \bar{X} \pm \Delta X$$

This equation implies that X value is between $\bar{X} - \Delta X$ and $\bar{X} + \Delta X$.

Relative error is given by

$$\text{Relative Error} = \frac{\Delta X}{\bar{X}}$$

and is described with a per cent %. Some quantities may have to be calculated by substituting the measured data in a formula. In this case, error propagation should be applied because an error might be present in each term in the formula.

Example 1.

$$\boldsymbol{r = x \pm y}$$

$$\Delta r = \Delta x + \Delta y$$

$$r = \left(x \pm y\right) \pm \Delta r$$

Example 2.

$$\boldsymbol{r = x.y}$$

$$\Delta r = \left(\Delta x\right) + \left(\Delta y\right)$$

$$r = \left(x.y\right) \pm \Delta r$$

$$\frac{\Delta r}{r} = \frac{\left(\Delta x\right) y + \left(\Delta y\right) x}{\left(x.y\right)} = \frac{\left(\Delta x\right)}{x} + \frac{\left(\Delta y\right)}{y}$$

Example 3.

$$\boldsymbol{r = \frac{x}{y}}$$

$$\Delta r = \frac{\left(\Delta x\right)}{y} + \frac{x\left(\Delta y\right)}{y^2}$$

$$r = \frac{x}{y} \pm \Delta r$$

$$\frac{\Delta r}{r} = \frac{\left(\Delta x\right)}{x} + \frac{\left(\Delta y\right)}{y}$$

SIGNIFICANT DIGITS

Sensitivity of a physical data may not always be given with its absolute error. It may be understood with its significant digits. The significant digits in a measurement can be used to have an idea about uncertainity. If, for example, the mass of an object is 26.4 g, the number of significant digits will be 3. If the mass is given as 0.0264 kg, the number of significant digits will not change. The zeros before a number are not significant. For example, there are 5 significant digits in 1.2468 and 2 significant digits in 0.00082. The sensitivity of a quantity increases with the number of significant digits.

Rounding Numbers

The results of a measurement should be rounded to the significant digit. The rules to round the numbers are as follows:

- If the number of last digit is smaller than 5 (0, 1, 2, 3, or 4), this digit is removed without changing the other digits.
- If the number of last digit is equal to 5 or greater than 5 (5, 6, 7, 8, or 9), this digit is removed and the value of the previous digit is increased by 1.

For example:

- 5.321 can be rounded up to 5.32, 5.3 or 5 depending on the required significant digits.
- 7.678 can be rounded up to 7.68, 7.7 or 8 depending on the required significant digits.

In addition or subsruction of numbers, the result can be rounded up with respect to the number which has the lowest decimal digits. For example,

$$2.2339 + 6.4 = 8.6339 = 8.6$$

$$12.12 - 8.4317 = 3.6883 = 3.69$$

In multiplication and dividing of numbers, similarly, the result can be rounded up with respect to the number which has the lowest decimal digits. For example,

$$4.4567 \times 3.56 = 15.865852 = 15.87$$

$$8.56 \times 0.00028 = 0.0023968 = 0.0024$$

GRAPHICS

Graphics show the relations between several quantities, parameters or measurable variables. A graph is basically a presentation of numerical relationship on how a quantity changes with another one that depends on the first quantity (changing during measurement). Generally, in fundamental physics experiments, two dimensional graphic representations are enough to show a relation between two variables related to each other. In this kind of graphs, the horizontal axis is independent variable and the vertical axis is dependent variable. Position-time graph of a moving object is given in Figure 3. Here, the quantity (dependent variable) will be analyzed with independent time variable because measurements are time-independent. In all graph plotting processes, the position data obtained from measurements must be defined with respect to the origin of the graph as a reference. The coordinates of any data are shown with a symbolization (x,y) in x-y plane with respect to the origin (0,0) in two dimensional graphs.

There is a linear relation between x and y variables and given by

$$y = mx + b$$

where m and b are constants. In a plotted x-y graph, m will be the slope of a straight line and corresponds to a physical quantity.

Consider two data points as $\left(x_1, y_1\right)$ and $\left(x_2, y_2\right)$ on a straight line of a graph. Difference between these two data points on x and y axes will be $\Delta x = x_2 - x_1$ and $\Delta y = y_2 - y_1$, respectively. The slope of the graph in Figure 3 is given by

$$m = \tan\theta = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}.$$

The procedure in a graph plotting process is given below:

- Title of the graph must be written.
- Labels of x and y axes must be shown clearly.
- The axes must be suitably scaled.
- Data points must be explained in detail.

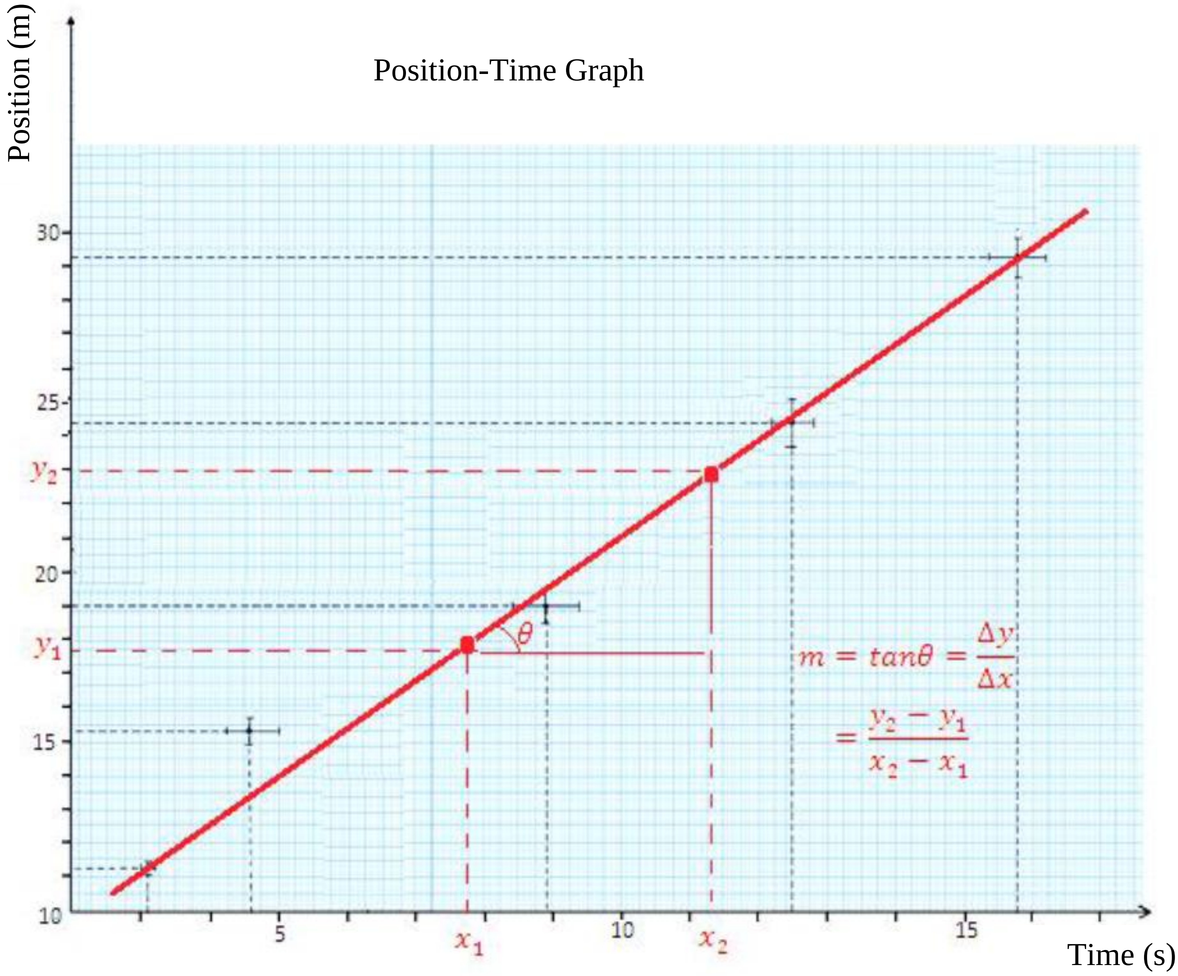


Figure 3. Position-time graph.

PROCEDURE:

EXPERIMENTAL KIT

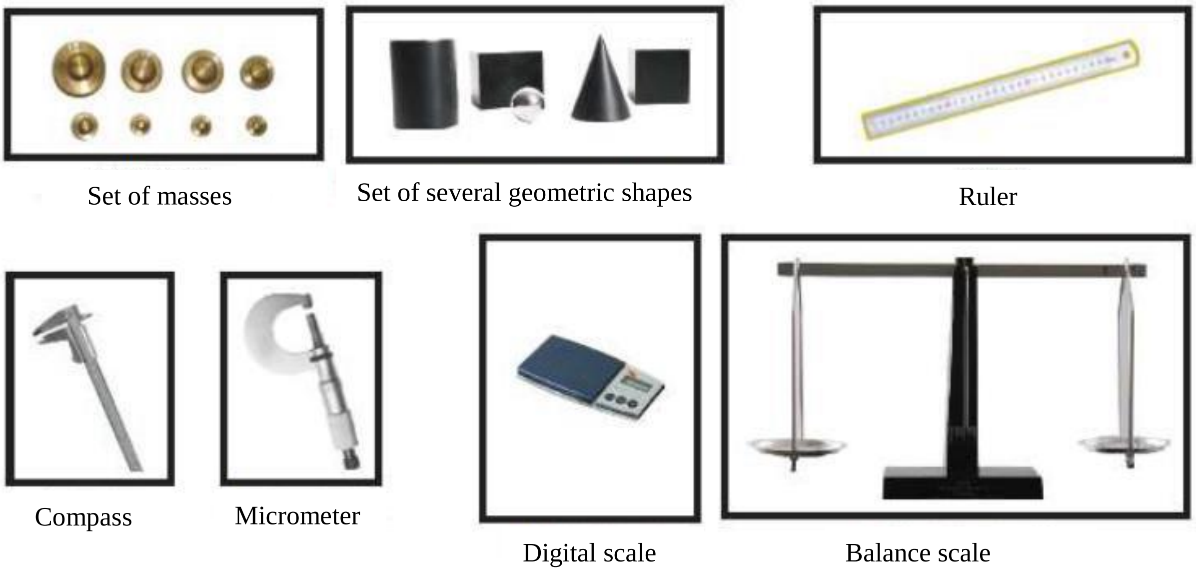


Figure 4. Equipments for the experiment.

RULER

A ruler is widely used as a measurement device for length measurements and to draw a straight line. The smallest interval on a ruler is 1 mm. The best estimation should be done when you read a measurement value between milimetric intervals. When you look at a ruler from the right or left side of the perpendicular vision angle, the position will be measured differently. This case is named as ***'parallax error'***. Figure 5 shows an example of the position of an object near a ruler.

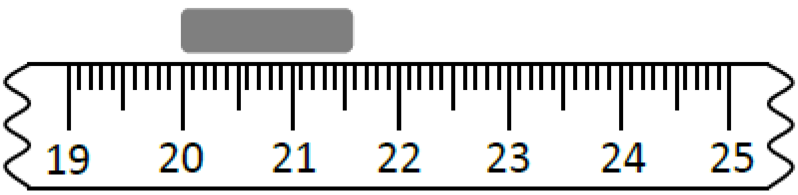


Figure 5. Length measurement of an object.

Two end points of the object can be read by using the ruler as shown in Figure 5. Line width of milimetric scales and parallax effect in observations must be considered during measurements. Therefore, error estimation limit is equal to the smallest scale interval, 1 mm. Hence, the positions of two end points on a ruler are 20.0 cm and 21.6 cm, respectively.

$$d_1 = 20.0 \pm 0.1 \text{ cm}$$

$$d_2 = 21.6 \pm 0.1 \text{ cm}$$

Here, d_1 and d_2 are the position measurement data for two end points of the object. The difference between these two end points is the length of the object:

$$d = d_2 - d_1 = (21.6 \pm 0.1 \text{ cm}) - (20.0 \pm 0.1 \text{ cm}) = 1.6 \pm 0.2 \text{ cm}$$

$$d = 16 \pm 2 \text{ mm.}$$

VERNIER CALIPER

A caliper is extensively used in laboratories and industry to take measurements very precisely. Contrary to a ruler, a caliper supplies exact and precise result on the decimal part of the measurement. Therefore, a caliper is a standard device for sensitive measurements. A caliper can be used to measure the length of an object, inner and outer radius of a hole and depth of a hole.

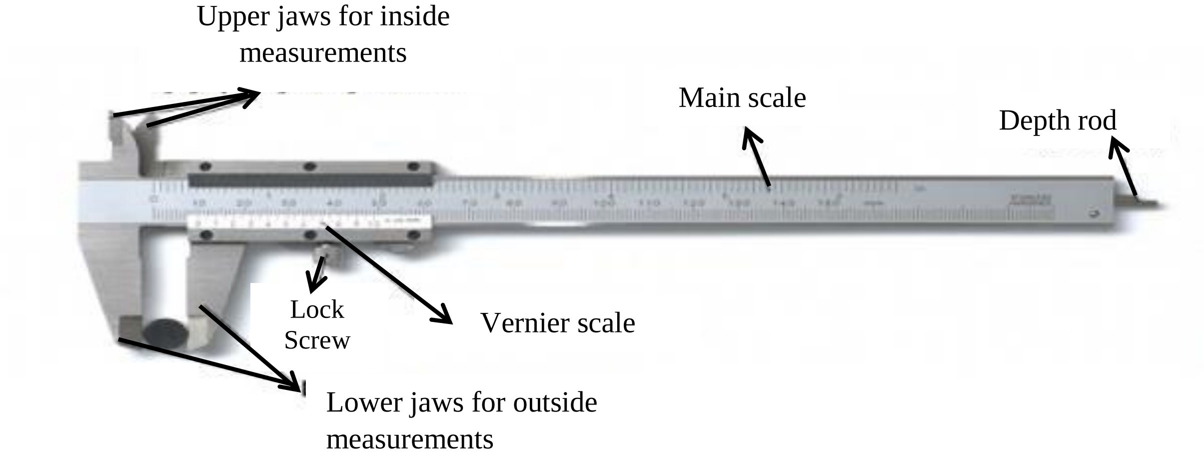


Figure 6. Caliper and its parts.

As seen in Figure 6, main scale, vernier scale, and jaws are main parts of a vernier caliper.

Vernier caliper is used for precise measurement in 1/10 mm (in some calipers 0.5/10 mm) scale. Vernier caliper is composed of a ruler with a millimeter scale and a vernier that can slide on the ruler. A vernier of 1/10 mm has a length of 9 mm which is divided into 9 equal parts. There is 0.1 mm between mm division of a caliper and the division of the vernier. During a measurement, first one needs to determine the location of the zero point of the vernier on the main scale; then the smaller value in the mm division is considered for the observation.

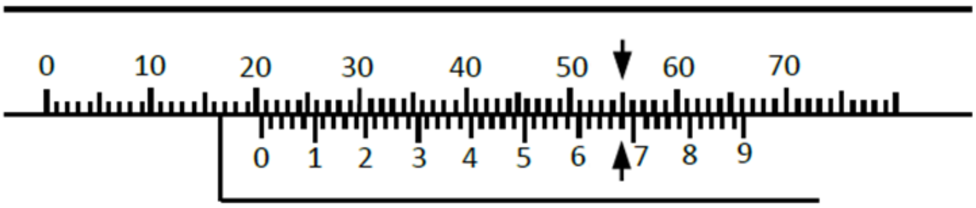


Figure 7. Reading a caliper.

Let's read the value in Figure 7. The zero point on the vernier is in between 20 and 21 on the main scale. Therefore, the correct reading on the main scale will be 20 mm. Now, we need to find the division line on the Vernier scale that exactly coincides with a division line on the main scale. This value on the Vernier is 6.8 and the resolution of the vernier is 0.1 (1/10). As a result, observed reading is $20\text{ mm} + 6.8 \times 0.1\text{ mm} = 20.68\text{ mm}$.

MICROMETER

Micrometer is a tool to measure smaller lengths in 1/100 mm precision. In Figure 8, a micrometer and its parts are shown.

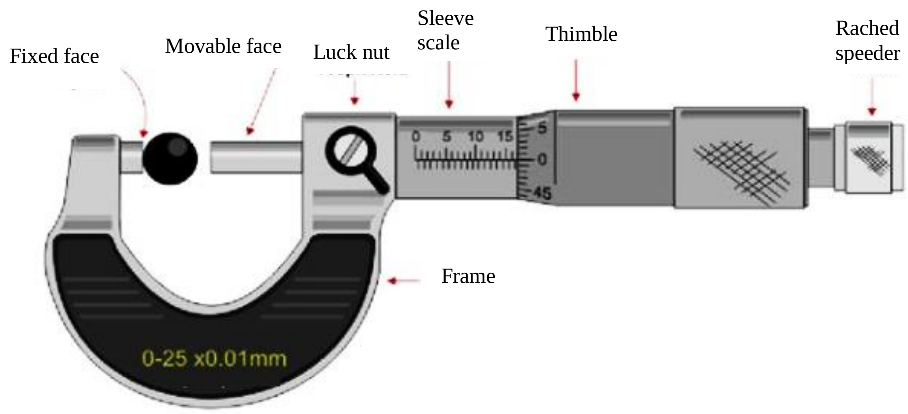


Figure 8. A micrometer and its parts.

In a micrometer, the sleeve part has a straight line that is scaled to show millimeters on its upper side and half millimeters on its lower side. There are 50 divisions on the thimble part (100 divisions in some micrometers). If the thimble has one complete turn, the jaws will have 0.5 mm separation (1 mm in some micrometers). If there is no separation between the jaws, zero line on the sleeve will coincide with the straight line on the thimble. Let's read the value in Figure 9.

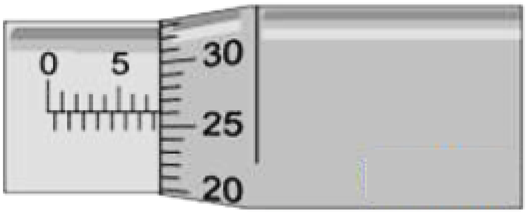


Figure 9. Reading a micrometer.

Passed complete divisions on the sleeve = 7 mm

Passed half division on the sleeve = 1 x 0.5 mm = 0.5 mm

Coincidence line on the thimble = 26 x 0.01 = 0.26 mm

Reading value = 7 + 0.5 + 0.26 = 7.76 mm

BALANCE SCALES

A balance scale is a widely used tool for mass measurement. It works with the principle of balancing a known mass with an unknown mas. A typical balance scale is shown in Figure 10.

Figure 10. A balance scale.

A balance scale consists of two pans hanged on the ends of a straight stick that is called as scale arm. The scale arm is placed on a sharp support from its center of mass. The distance from the scale arm to the support is called as **'the support point'**. The sum of the known masses put on a pan to balance an unknown mass that is on the other pan gives the value of the unknown mass. In most balance scales, there are calibrated small masses on the scale arm which can freely move on the arm and provide precise measurements. In these measurement devices, accuracy of the observed data depends on the sensitivity of these small masses.

A balance scale must be checked before taking any measurement whether its needle connected to the scale arm shows zero point on the support arm. A perfect balance to have 100 % accuracy is physically impossible. Accuracy in a balance scale is easily affected by humidity and dust particles on the pans. Careful observation during a measurement is important for the accuracy of the results.

DIGITAL SCALES

Mass measurement of an object with a digital scale is the easiest and the fastest way. The digital scales are used more often to obtain more accurate and trustworthy results very fast and easily. Also, a mass can be measured with several different units such as gram and ounce by using the digital scales. Therefore, the unit that will be used should be introduced to the device when it is turned on. When the object is placed on the scale disc, the mass of the object will be shown on the digital screen. There is no uncertainty in reading values of a digital scale, but sensitivity of the device must be known.

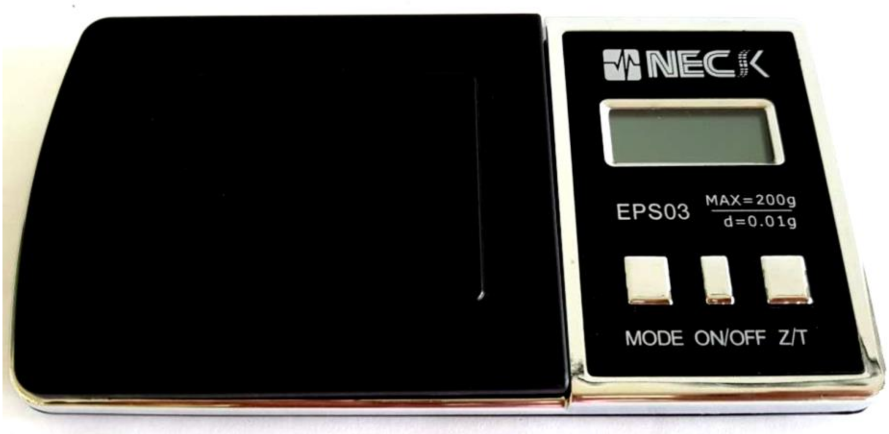


Figure 11. A digital scale.

PROCEDURE:

The densities of some given objects shown in Figure 12 will be calculated.

Figure 12. Objects with different geometry.

In order to determine the masses of the objects, a balance scale and a digital scale will be used. The dimensions of the bodies will be measured by a caliper, a micrometer, and a scale, and volumes of the objects will be calculated using these data.

$$V_{\text{rectangle}} = WLh$$

$$V_{\text{cylinder}} = \pi r^2 L$$

$$V_{\text{cone}} = \frac{1}{3}\pi r^2 h$$

$$V_{\text{sphere}} = \frac{4}{3}\pi r^3$$

Here;

***W*, *L*, and *h*:** The width, the length, and the height of the rectangle

***r* and *L*:** The radius and the length of the cylinder

***r* and *h*:** The base radius and the height from the base to the apex cone

***r*:** the radius of the sphere.

Let us consider briefly the concept of density before the experiment. Density is a characteristic feature of a material and is independent of the amount of material. Thus, the density remains the same regardless of its shape and size. A mass can be distributed into its volume regularly (homogeneous) or irregularly (nonuniform). If a mass is regularly distributed, the body density ρ is calculated by dividing the total mass (m) by the total volume (V):

$$\rho = \frac{m}{V}$$

SI unit system gives kgm^{-3} for density. However, gcm^{-3} is used more widely in defining the density of an object.

The procedure to follow in this experiment is given below:

1. Measure the dimensions of the rectangle (W, L, h) with a ruler, the dimensions of the cylinder and the cone ($r_{cylinder}$, L, r_{cone}, h) with a caliper and the radius of the sphere $\left(r_{sphere}\right)$ with a micrometer. Record these values in Table 3.
2. Use the appropriate volume formula of each geometric object to calculate their volumes ($V_{rectangle}$, $V_{cylinder}$, V_{cone}, V_{sphere}) and record them in Table 3.
3. Measure the masses of all geometric objects ($m_{rectangle}$, $m_{cylinder}$, m_{cone}, m_{sphere}) using a scale and record them in Table 3.
4. Calculate the density of each geometric object ($\rho_{rectangle}$, $\rho_{cylinder}$, ρ_{cone}, ρ_{sphere}) and record them in Table 3. Interpret your results.

EXPERIMENTAL DATA:

TABLE 3			
Rectangle	Cylinder	Cone	Sphere
$W=$ $L=$ $H=$	$r_{cylinder}=$ $L=$	$r_{cone}=$ $h=$	$r_{sphere}=$
$V_{rectangle}=$	$V_{cylinder}=$	$V_{cone}=$	$V_{sphere}=$
$m_{rectangle}=$	$m_{cylinder}=$	$m_{cone}=$	$m_{sphere}=$
$\rho_{rectangle}=$	$\rho_{cylinder}=$	$\rho_{cone}=$	$\rho_{sphere}=$

MOTION WITH CONSTANT ACCELERATION AND CONSERVATION OF LINEAR MOMENTUM

M1

MOTION WITH CONSTANT ACCELERATION AND CONSERVATION OF LINEAR MOMENTUM

M1

PURPOSE OF THE EXPERIMENT:

1. To study the motion with constant acceleration in an inclined plane.

2. To investigate the conservation of linear momentum on a collision between two objects in two dimensions.

EQUIPMENT FOR THE EXPERİMENT:

Air table set

Arc chronometer and foot pedal

Air compressor and foot pedal

Discs

Carbon paper

THEORY:

1.1. MOTION WITH CONSTANT ACCELERATION

If the net force acting on an object moving in a uniform linear path is zero, then the object keeps its motion with constant velocity. When the object is assumed to be at an initial position of $\vec{x} = \vec{x}_0$, since there is no change in velocity, the position of the object at a time t is given by

$$\vec{x} = \vec{x}_0 + \vec{V}t \tag{1.1}$$

and if the initial position is $\vec{x}_0 = \vec{0}$ then

$$\vec{x} = \vec{V}t \tag{1.2}$$

is obtained where $\vec{V}$ is the velocity of the object in ms^{-1} in SI unit system. In a uniform linear motion with constant velocity, an object moves equal distances in equal time intervals on a linear path.

If the velocity or the direction of the object or both changes, the object will accelerate or decelerate. As shown in Figure 1.1, as an object slides down an inclined frictionless plane a constant acceleration motion occurs. The magnitude of the acceleration of the object depends on the inclination angle (θ) with the horizontal plane.

If an object is allowed to slide down from the highest point of an inclined frictionless plane, the object moves along a linear path with an increasing velocity. The rate of the change in the object velocity gives the acceleration of the object.

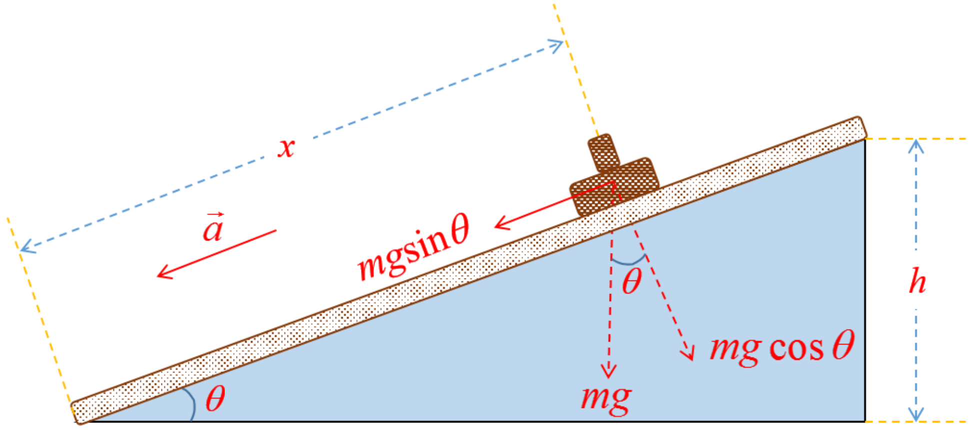


Figure 1.1. An object sliding down on a frictionless inclined plane.

The average acceleration of the object in a time interval of Δt is defined as

$$\vec{a}_{ort} = \frac{\Delta \vec{V}}{\Delta t} = \frac{\vec{V}_2 - \vec{V}_1}{t_2 - t_1} \tag{1.3}$$

where $\vec{V}_1$ is the velocity of the object at time t_1 and position x_1; $\vec{V}_2$ is the velocity of the object at time t_2 and position x_2, and $\Delta \vec{V}$ is the change in the velocity of the object between positions x_1 and x_2. The unit of acceleration is ms^{-2} in the SI system.

The instantaneous acceleration of an object at any time *t* is given by

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{V}}{\Delta t} = \frac{d\vec{V}}{dt} \cdot \tag{1.4}$$

In a uniform linear motion, instantaneous acceleration is equal to the change of velocity at any given time.

Assuming that initial time is $t = 0$, initial velocity of the object is $\vec{V}_0$, and initial position is $\vec{x}_0$, the velocity and position of the object at any given time for a linear motion with constant acceleration are given by

$$\vec{V} = \vec{V}_0 + \vec{a}t \tag{1.5}$$

$$\vec{x} = \vec{x}_0 + \vec{V}_0 t + \frac{1}{2}\vec{a}t^2 \cdot \tag{1.6}$$

If an object starts from rest to move from an initial position $\vec{x}_0 = \vec{0}$, its position at any given time t is obtained as;

$$\vec{x} = \frac{1}{2}\vec{a}t^2 \cdot \tag{1.7}$$

According to Equation 1.7, plotting a graph of $x \sim t^2$ yields a linear line, and the slope of this line is expressed as

$$m = \tan\theta = \frac{1}{2}a \cdot \tag{1.8}$$

Thus, the acceleration a of the object can be determined experimentally.

Figure 1.1 shows an object sliding down on an inclined air table. If a free-body diagram is drawn for this object, it can be seen that the magnitude of force that acts upon the object in the direction of the motion is $F = mg\sin\theta$. According to Newton's second law of motion the equation $F = ma$ is valid for constant mass. Thus, the acceleration of the object is obtained as

$$F = mg\sin\theta = ma \tag{1.9}$$

$$a = g\sin\theta \tag{1.10}$$

and this is the horizontal component of gravitational acceleration. If the inclination angle θ is known and the acceleration of the object is determined, the gravitational acceleration can be found experimentally on that point. At $\theta = 90°$, the acceleration of the object will be equal to the gravitational acceleration. If the inclination angle θ and the gravitational acceleration g are known, the acceleration of the object can be calculated by using Equation 1.10.

1.2. CONSERVATION OF LINEAR MOMENTUM

The momentum of an object with mass m and velocity $\vec{V}$ is given by

$$\vec{P} = m\vec{V} \cdot \tag{1.11}$$

The direction of the momentum vector is the same as the direction of motion. Its unit is given as $kgms^{-1}$ in SI unit system with mass kg and velocity ms^{-1}.

If the net force acting on an object is zero, its momentum remains constant. This is known as "***Conservation Law of Momentum***". This law is one of the fundamental conservation laws of the universe.

Newton's second law of motion for an object in terms of momentum is given by

$$\vec{F}_{net} = \frac{d\vec{P}}{dt} \cdot \tag{1.12}$$

According to Equation 1.12, the change of momentum over time gives the net force that acts on the object. If the net force that acts on a constant mass is zero, the total momentum is conserved, namely remains constant.

$$\text{If } \vec{F}_{net} = \vec{0} \text{ then } \vec{P} = \text{constant} \quad (\textbf{\textit{Conservation Law of Momentum}}) \tag{1.13}$$

According to Equation 1.13, it is obvious that the initial momentum of the object is equal to its final momentum. Since momentum is a vector quantity, the conservation of momentum should be written by considering all components of the momentum of an object.

Let's assume that a force $\vec{F}$ acts on an object with mass *m* and velocity $\vec{V}_1$ over a short time Δt and consequently the velocity of the object becomes $\vec{V}_2$. According to Newton's second law of motion

$$\vec{F} = \frac{m\left(\vec{V}_2 - \vec{V}_1\right)}{\Delta t}$$

$$\vec{F}\Delta t = m\left(\vec{V}_2 - \vec{V}_1\right) = m\vec{V}_2 - m\vec{V}_1 = \vec{P}_2 - \vec{P}_2 = \Delta\vec{P} \tag{1.14}$$

is obtained. The quantity $\vec{F}\Delta t$ is called as ***"impulse"*** that is the integral of a force $\vec{F}$ acting on an object over a time interval Δt. As seen in Equation 1.14, the impulse on an object is equal to the momentum change. The unit of the impulse in SI unit system is $kgms^{-1}$ or Ns.

A short-duration interaction between two bodies or more than two bodies simultaneously at the intersection of their paths is called as "**collision**". There are two types of collisions: elastic collision and inelastic collision. In both types of collisions the momentum is conserved. In other words, the momentum before the collision is equal to the momentum after the collision. In an elastic collision, the total kinetic energies before and after the collision are equal to each other. A collision where the kinetic energy is not conserved is called as "inelastic collisions". The kinetic energy loss is converted to other form of energy mostly to heat and as a result, the total energy is always conserved.

If the friction is neglected, for a two-body system the total momentum before the collision $\left(\vec{P}_{ini}\right)$ will be equal to the total momentum after the collision $\left(\vec{P}_{fin}\right)$:

$$\vec{P}_{ini} = \vec{P}_{fin}$$

$$m_1\vec{V}_{1_{ini}} + m_2\vec{V}_{2_{ini}} = m_1\vec{V}_{1_{fin}} + m_2\vec{V}_{2_{fin}} \tag{1.15}$$

$\vec{V}_{1_{ini}}$: velocity of mass m_1 before the collision

$\vec{V}_{2_{ini}}$: velocity of mass m_2 before the collision

$\vec{V}_{1_{fin}}$: velocity of mass m_1 after the collision

$\vec{V}_{2_{fin}}$: velocity of mass m_2 after the collision.

When the two bodies are identical ($m_1 = m_2 = m$), the momentum conservation in Equation 1.15 is reduced to

$$\vec{V}_{1_{ini}} + \vec{V}_{2_{ini}} = \vec{V}_{1_{fin}} + \vec{V}_{2_{fin}} \tag{1.16}$$

In an elastic collision, the kinetic energy of each body may change, but the total kinetic energy does not change; that is, it is conserved. However in an inelastic collision, the kinetic energy of the system is not conserved.

In an elastic collision of two bodies, the total kinetic energy (K_{ini}) before the collision is equal to the total kinetic energy (K_{fin}) after the collision, which is called as conservation of kinetic energy:

$$K_{ini} = K_{fin} \tag{1.17}$$

$$\frac{1}{2} m_1 V_{1_{ini}}^2 + \frac{1}{2} m_2 V_{2_{ini}}^2 = \frac{1}{2} m_1 V_{1_{fin}}^2 + \frac{1}{2} m_2 V_{2_{fin}}^2 \tag{1.18}$$

In an inelastic collision the momentum is conserved, but the kinetic energy is not. In this case, **"proportional loss"** of the kinetic energy is defined as

$$\text{Proportional loss} = \frac{K_{ini} - K_{fin}}{K_{ini}} \tag{1.19}$$

and **"percentage loss"** is expressed as

$$\text{Percentage loss} = \frac{K_{ini} - K_{fin}}{K_{ini}} \times 100. \tag{1.20}$$

PROCEDURE:

Experimental set-up is shown in Figure 1.2.

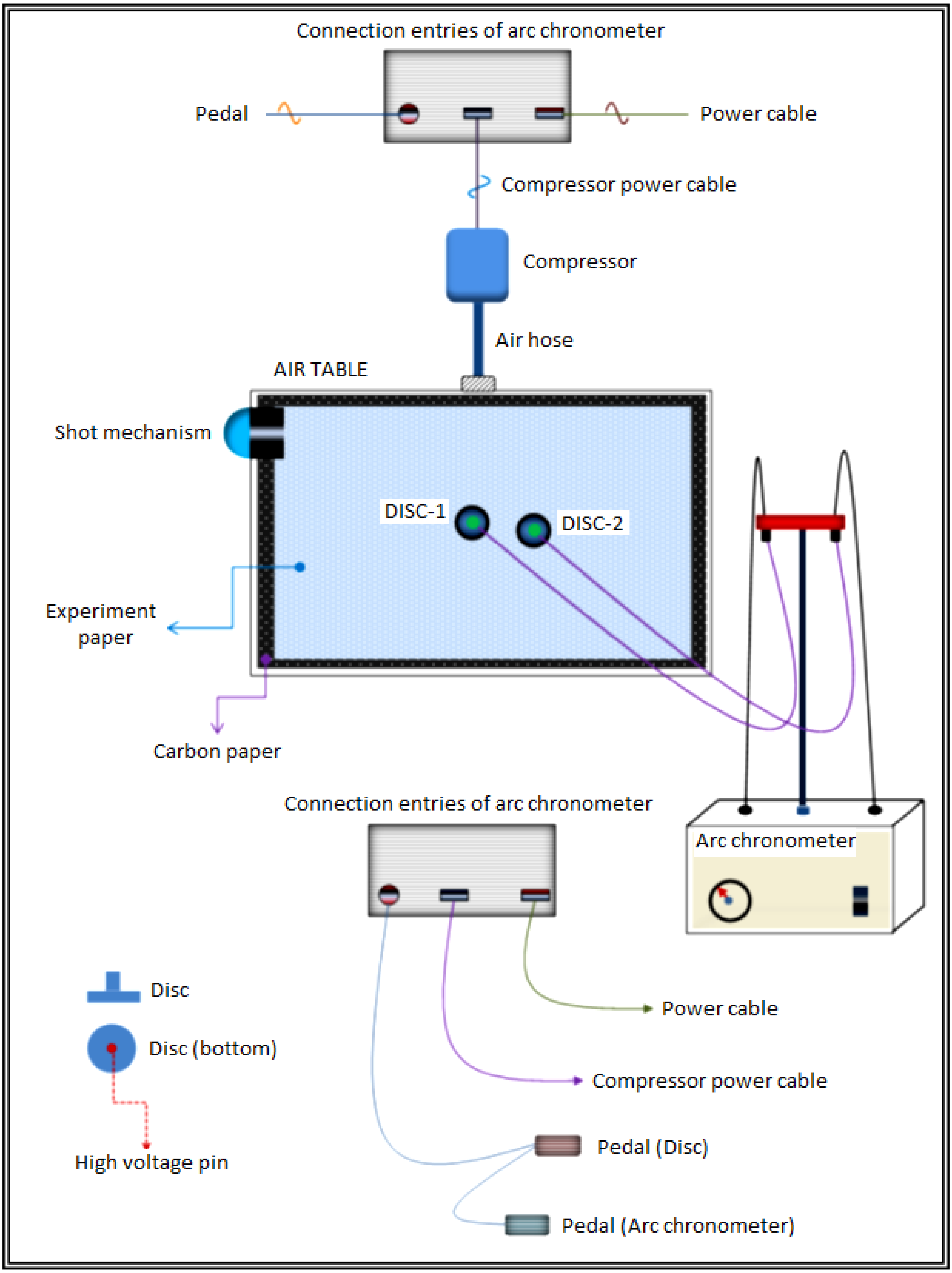


Figure 1.2. Schematic drawing of the experimental air table set.

A. Motion with Constant Acceleration

A1. Place the experiment paper onto the carbon paper located on the air table.

A2. Set the air table to align horizontally and then elevate one end of the air table by a wooden block to make a certain inclination angle θ.

A3. Put one of the metal discs onto the carbon paper at the lower-right corner of the air table.

A4. Turn on the compressor. Put the next metal disc onto the top of the inclined plane and push the compressor pedal. Check out if the disc is sliding down freely.

A5. Turn on the arc chronometer and set the frequency to $f = 20$ Hz.

Do not touch the conductive sections, high voltage ends, metal discs hoses, experiment paper or carbon paper when the arc chronometer is in progress.

A6. Place the metal disc at the middle point of the upper end of the air table.

- Press the arc chronometer pedal and air compressor pedal simultaneously, and release the metal disc from rest.
- When the disc reaches the lower end of the inclined plane, release the pedals.
- Observe that the disc is moving in a straight path.
- Turn off the arc chronometer.

A7. Remove the experiment paper.

- Take the positive x-axis as the direction of the motion for the metal disc.
- Starting from the first point, enumerate the points as 0, 1, 2,…, 10.
- Assume the first datum point as $x = 0$ and $t = 0$.
- Measure the distance (x) between the initial point and the following every point by using a ruler, and record it in Table 1.1.
- Determine the time (t) for each point and record it in Table 1.1.

Because the arc chronometer's frequency is set to $f = 20$ Hz, the metal disc generates 20 points per second on the experiment paper. Therefore, the time interval between two sequential points will be 1/20=0.05 s.

- Calculate the values t^2 and record them on Table 1.1.

A8. Draw the $x = f\left(t^2\right)$ graph and obtain a linear line for the data. Determine the experimental acceleration (a) from the slope of this line by using Equation 1.8 and record it in Table 11. Show the calculations clearly on the chart.

A9. Calculate the expected acceleration by using Equation 1.10, record it in Table 1.1 and compare it with the experimental acceleration (a') ($g = 9.8\,\mathrm{ms}^{-2}$).

B. Conservation of Linear Momentum

B1. Adjust the legs of the air table to align horizontally.

B2. Turn on the compressor and slide the two metal discs for collision to each other diagonally. Repeat this process several times until you get a satisfying collision.

B3. Turn on the arc chronometer and set the frequency to $f = 20\,\mathrm{Hz}$.

B4. Push the arc chronometer pedal and air compressor pedal simultaneously, and slide the two metal discs to each other diagonally for collision near the middle of the air table.

B5. Release the pedals after collision.

B6. Turn off the arc chronometer and pick up the metal discs from the air table for safety.

B7. Remove the experiment paper.

- The points of the metal discs must be as shown in Figure 1.3.
- Enumerate the points on the experiment paper for each metal disc as 0, 1, 2,…, 10.
- As given in Figure 1.3, label the paths that are produced by metal discs as A, B for before the collision and as A’, B’ for after the collision.

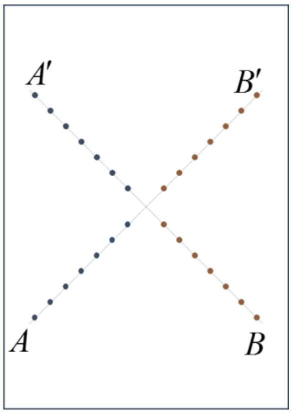
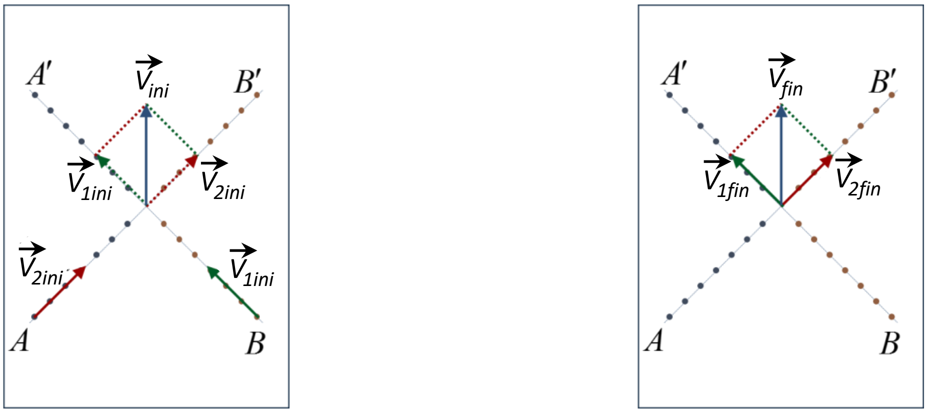


Figure 1.3. Paths of metal discs before and after the collision.

B8. Determine the magnitude of the velocity before and after the collision for two metal discs by measuring two or three intervals and divide it by the time for each path and record it in Table 1.2.

B9. Obtain the vector sum $\vec{V}_{ini} = \vec{V}_{1_{ini}} + \vec{V}_{2_{ini}}$.

- In order to determine the vector sum, lengthen the paths A and B until they cross each other as shown in Figure 1.4 (a).



(a) **(b)**

Figure 1.4. The vector sum of velocities **(a)** before and **(b)** after the collision.

- Draw the velocity vectors $\vec{V}_{1_{ini}}$ and $\vec{V}_{2_{ini}}$ on the graph paper by starting from the junction point. Be careful to draw the lengths of these velocity vectors to be proportional to their magnitudes.
- Obtain the vector sum $\vec{V}_{1_{ini}} + \vec{V}_{2_{ini}}$ before the collision by using parallelogram method, calculate its magnitude and record it in Table 1.2.

B10. As shown in Figure 1.4 (b), obtain the vector sum $\vec{V}_{fin} = \vec{V}_{1_{fin}} + \vec{V}_{2_{fin}}$, the magnitude of this vector, and record it in Table 1.2.

B11. Compare the vector sums for velocities before and after the collision, and comment on your result.

B12. Obtain the total kinetic energies before and after the collision. Record it in Table 1.2 and comment on your result.

EXPERIMENTAL DATA:

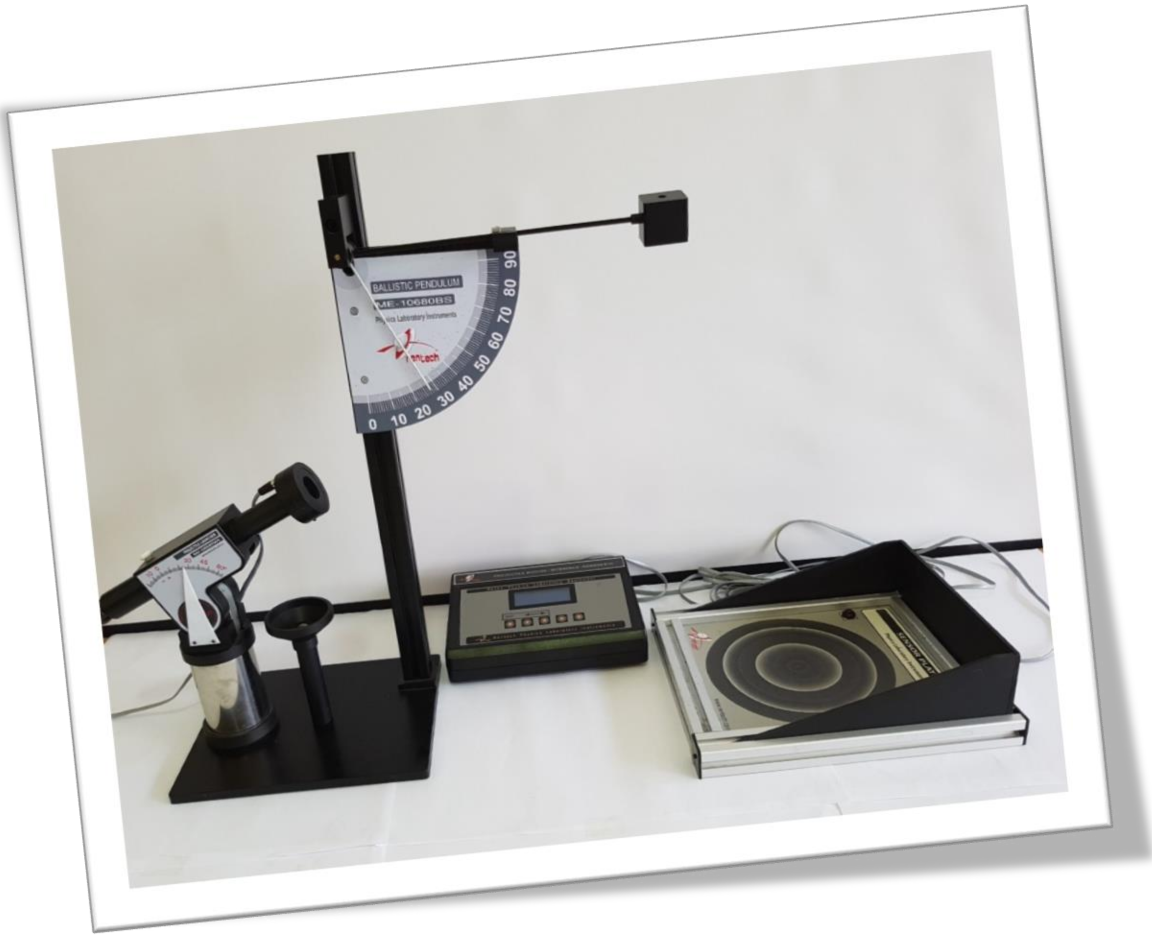
TABLE 1.1

Inclination Angle (θ)	Frequency (Hz)	Point Number	x (m)	t (s)	t^2 (s^2)	a (ms^{-2})	a' (ms^{-2})
						Experimental	Expected

TABLE 1.2

$V_{1_{ini}}$ (ms^{-1})	$V_{2_{ini}}$ (ms^{-1})	$V_{1_{fin}}$ (ms^{-1})	$V_{2_{fin}}$ (ms^{-1})	$\vec{V}_{1_{ini}} + \vec{V}_{2_{ini}}$	$\vec{V}_{1_{fin}} + \vec{V}_{2_{fin}}$	$\left\lvert\vec{V}_{1_{ini}} + \vec{V}_{2_{ini}}\right\rvert$	$\left\lvert\vec{V}_{1_{fin}} + \vec{V}_{2_{fin}}\right\rvert$	K_{ini} (J)	K_{fin} (J)

BALLISTIC PENDULUM
AND
PROJECTILE MOTION



M2

BALLISTIC PENDULUM AND PROJECTILE MOTION

M2

PURPOSE OF THE EXPERIMENT:

1. To determine the initial velocity of a projectile (steel ball) by using a ballistic pendulum.

2. To investigate the relationship between kinetic energy and potential energy in a ballistic pendulum system.

3. To learn the principles of conservation of momentum and conservation of mechanical energy.

4. To understand the kinematic equation of motion in two dimensions.

5. To obtain the initial velocity of a projectile launched horizontally by analyzing the projectile motion.

6. To predict and verify the horizontal distance travelled by a projectile launched at an angle above the horizontal.

EQUIPMENT FOR THE EXPERIMENT:

Ballistic pendulum

Projectile launcher

Photogate timer

Force plate (Timer plate)

Photogate

Steel ball

Ruler

White paper and carbon copying paper

THEORY:

2.1. BALLISTIC PENDULUM

The ballistic pendulum is a device to measure the horizontal velocity (initial velocity) of a fast projectile, such as a bullet. The ballistic pendulum consists of a rod positioned in a perpendicular position which can rotate around its frictionless pivot point and a block (catcher).

As seen in Figure 2.1 (a), the projectile with mass *m* is fired with initial horizontal velocity $\vec{V}$ towards the ballistic pendulum which is initially at rest. In this pendulum system, the horizontally thrown projectile is a steel ball of mass *m*. The ball collides with the pendulum, as a result of this collision, ballistic pendulum and the ball with mass m start to move together with velocity $\vec{V}'$ in the forward direction, as shown in Figure 2.1 (b). In this case, the ball gets stuck in the block of mass M in the ballistic pendulum, and an inelastic collision takes place. By the collision, the pendulum and the ball together swing up to a maximum height h as seen in Figure 2.1 (c). The angle θ corresponding to maximum height to which the pendulum swings is determined by the experimental setup. The horizontal initial velocity of the ball can be calculated by using the principles of conservation of mechanical energy and conservation of momentum.

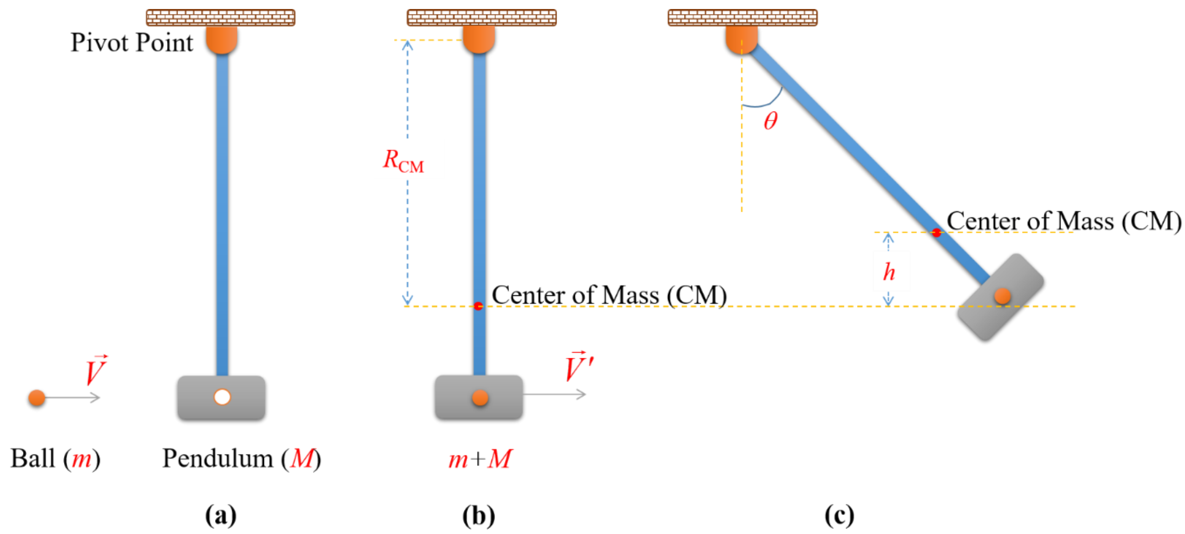


Figure 2.1. The states of the steel ball of mass m and the ballistic pendulum of mass M at the instants of **(a)** before collision, **(b)** right at collision and **(c)** after collision.

At first, we can consider the collision event seen in Figure 2.1 (b). We assume the collision time is very short such that the ball with mass m comes to rest in the block with mass M, before the block has moved from its rest position. In other words, the ball is embedded in the block, and the system behaves as a single object of mass $m+M$ moving with velocity $\vec{V}'$. In this inelastic collision event, momentum is conserved:

$$\vec{P}_{initial} = \vec{P}_{final}$$

$$m\vec{V} = (m+M)\vec{V}' \tag{2.1}$$

m **:** Mass of the projectile (ball)

$\vec{V}$ **:** Initial horizontal velocity of the projectile (ball) just before the collision

M **:** Mass of the pendulum block

$\vec{V}'$ **:** Velocity of the pendulum-ball system right after the collision

According to Equation 2.1, the linear momentum of the ball right before the collision is equal to the linear momentum of the pendulum-ball system *(m+M)* right after the collision.

As seen in Figure 2.1 (c), right after the collision, the pendulum and ball together act as a point mass located at their combined center of mass (CM) moving under the influence of gravity, which tends to pull it back to the vertical position. Therefore conservation of momentum cannot be used after the collision, however since gravity is a conservative force, conservation of mechanical energy can be used. The kinetic energy (*K*) after the collision is changed entirely into the gravitational potential energy (*U*) when the system's center of mass reaches its maximum height (*y*=*h*).

The mechanical energy (*E*) of an object or a system is the sum of its potential energy (*U*) and the kinetic energy (*K*). If the forces acting on an object are only conservative forces, the object's kinetic and potential energies can change during the motion; however, its mechanical energy remains constant. This is called principle of "***conservation of mechanical energy***". Therefore, according to the principle of conservation of energy, the system's total mechanical energy remains constant if the energy loss due to the friction can be neglected, in other words, mechanical energy is conserved:

$$E = K + U = \text{constant} \tag{2.2}$$

Kinetic energy is the energy associated with the state of motion of an object, the kinetic energy of an object of mass *m* moving with speed $\vec{V}$ is defined as;

$$K = \frac{1}{2}mV^2. \tag{2.3}$$

The gravitational potential energy of an object with mass *m* at any point of a vertical height *h* above a reference point can be defined as;

$$U = mgh. \tag{2.4}$$

The gravitational potential energy is the energy due to the object's or system's position, and its unit in SI unit system is Joule (J).

When the kinetic and potential energy terms are written in Equation 2.2, the mechanical energy of the system at any moment will be

$$E = \frac{1}{2}mV^2 + mgh \tag{2.5}$$

According to the principle of conservation of mechanical energy, if the kinetic energy of the object increases during its motion, its potential energy must decrease by the same amount, and vice versa. So, the total mechanical energy of the system remains constant.

As seen in Figure 2.1 (c) after the ball collides with the pendulum, center of mass of the combined pendulum-ball system swings up to the maximum height of $y=h$. In this case, the kinetic energy of the pendulum-ball system is converted into the potential energy. In other words, this potential energy is equal to the kinetic energy of the system ($m+M$) right after the collision. So, the conservation of mechanical energy can be written as

$$\frac{1}{2}(m+M)V'^2 = (m+M)gh \tag{2.6}$$

where h is the maximum change in the vertical position of the center of mass of the pendulum-ball system and g is the gravitational acceleration. Using Equation 2.6, the velocity of the pendulum-ball system right after the collision V' can be found as

$$V' = \sqrt{2gh} \tag{2.7}$$

and substituting this result in Equation 2.1, the initial horizontal velocity of the ball can be obtained as

$$V = \left(\frac{m+M}{m}\right)\sqrt{2gh}. \tag{2.8}$$

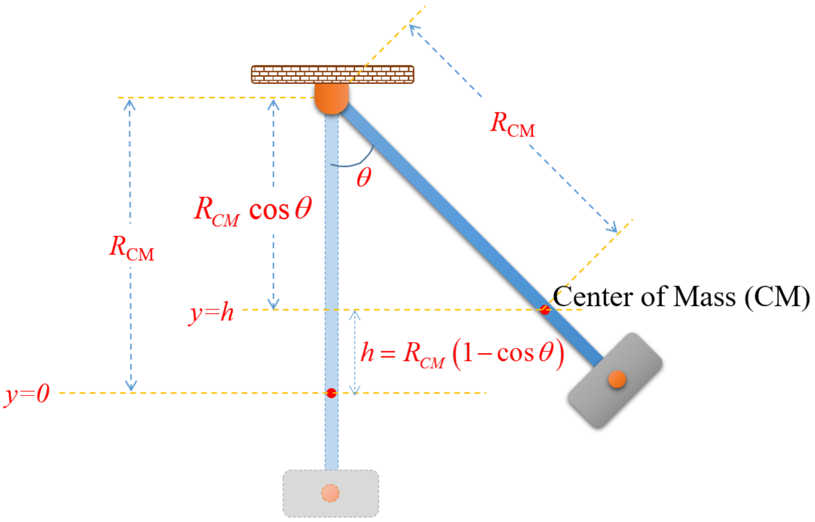


Figure 2.2. A state of the ballistic pendulum after the collision.

If the maximum angle through which the pendulum-ball system swings after the collision is θ at maximum height h as seen in Figure 2.2, using the maximum angle and the length of the pendulum from the center of mass to the pivot point R_{CM}, the maximum height (h) is given by

$$h = R_{CM}(1-\cos\theta). \tag{2.9}$$

When the Eqs. 2.8 and 2.9 are combined, the magnitude of the initial horizontal velocity of the ball is found as

$$V = \left(\frac{m+M}{m}\right)\sqrt{2gR_{CM}\left(1-\cos\theta\right)} \tag{2.10}$$

In the experiment, the maximum angle θ of the pendulum-ball system at the maximum height *h* after the collision is measured by using the angle indicator, and Equation 2.10 can be used to obtain the initial velocity of the ball before the collision.

2.2. HORIZONTAL PROJECTILE MOTION AND PROJECTILE MOTION

"***Projectile motion***" is a two-dimensional motion of an object under the influence of Earth's gravitational acceleration at the position of the object. An object fired from the projectile launcher with a specific initial velocity follows a path determined entirely by the effect of gravitational acceleration. The path followed by the object is called its "***trajectory***". The magnitude of the gravitational acceleration is

$$g = 9.8\ \mathrm{ms}^{-2} \tag{2.11}$$

Gravitational acceleration is taken to be constant during the motion, and its direction is toward the Earth's center.

When the object is fired from the launcher at a specific angle (θ) with horizontal, depending upon the value of angle θ, it is a horizontal projectile motion if $\theta = 0^\circ$, it is a vertical projectile motion if $\theta = 90^\circ$, and it is a projectile motion if $0^\circ < \theta < 90^\circ$. In this section, horizontal projectile motion, projectile motion, and horizontal range subjects will be investigated.

2.2.1. Horizontal Projectile Motion

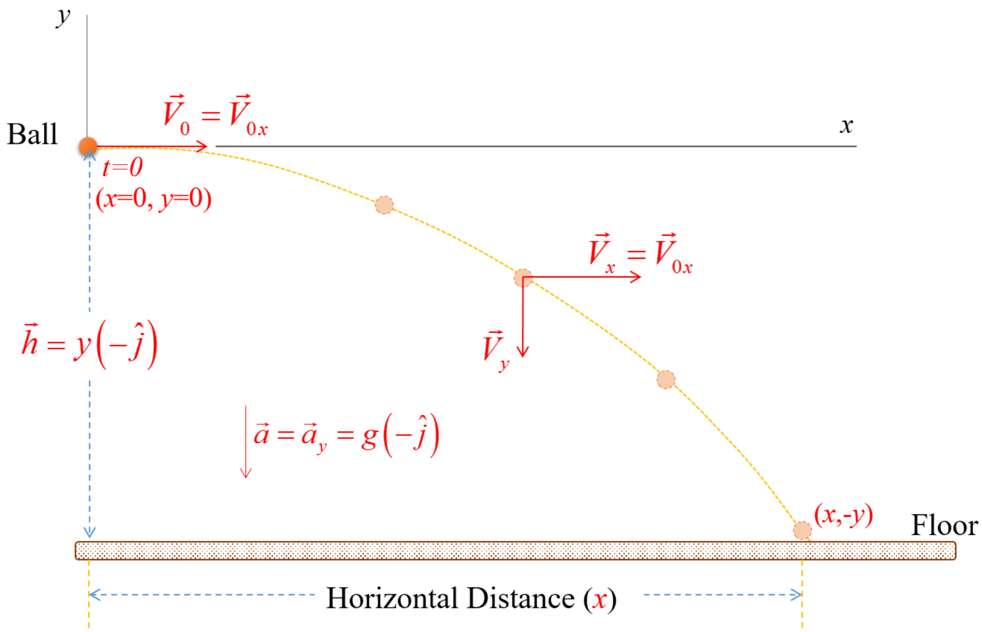


Figure 2.3. Horizontal projectile motion.

The trajectory of an object fired horizontally with an initial velocity $\vec{V}_0$ at $t = 0$ is shown in Figure 2.3. The dashed line shows the followed path under the influence of gravitational acceleration. In horizontal projectile motion, the object's initial velocity has only an x-component $(\vec{V}_0 = \vec{V}_{0x})$. Once the ball leaves the launcher, it experiences a vertically downward acceleration due to gravity ($\vec{g}$). So, a horizontally projected object's velocity has x and y components at any time t. Since there is not any force acting on the object in x direction, x-component of the object's velocity $(\vec{V}_x)$ is constant during the motion. Therefore, the acceleration of the object along the x-axis is zero $(\vec{a}_x = \vec{0})$ and a uniform linear motion is observed. y-component of the object's velocity $(\vec{V}_y)$, on the other hand, changes as a function of time under the influence of the gravitational force and thus, a motion with constant acceleration is observed. Along with these explanations acceleration vector becomes

$$\vec{a} = a_x\hat{i} + a_y\hat{j} = -g\,\hat{j} \tag{2.12}$$

$$a_x = 0$$

$$a_y = g = \text{constant}$$

Alternatively, the velocity vector at any time t is given by

$$\vec{V} = V_x\hat{i} + V_y\hat{j} \tag{2.13}$$

$$V_x = V_{0x} \tag{2.14}$$

$$V_y = V_{0y} - gt. \tag{2.15}$$

In the horizontal projectile motion, the object does not move just in the y-axis, instead, it moves on the xy-plane. In this case, according to Figure 2.3, the position vector of the object at any time t is given by

$$\vec{r} = x\hat{i} + y\hat{j} \tag{2.16}$$

$$x = x_0 + V_{0x}t \tag{2.17}$$

$$y = y_0 + V_{oy}t - \frac{1}{2}gt^2. \tag{2.18}$$

If the positive y direction is chosen downward, the negative (-) sign before the gravitational acceleration g in Eq. 2.18 should be (+).

As seen in Figure 2.3, if the origin of the x-y coordinate system is taken to be the object's initially launched position, at the time $t = 0$; then $x_0 = 0$ and $y_0 = 0$. For the object launched horizontally ($\theta = 0^\circ$), its initial velocity becomes horizontal and the vertical component of the velocity is zero:

$$V_{0x} = V_0 \tag{2.19}$$

$$V_{0y} = 0. \tag{2.20}$$

According to Eqs. 2.14 and 2.15, the object's horizontal and vertical velocity components (V_x and V_y) at any time t are given by

$$V_x = V_{0x} = V_0 = \text{constant} \tag{2.21}$$

$$V_y = V_{oy} - gt = 0 - gt = -gt \tag{2.22}$$

It is noticed that the magnitude of the velocity vector (V) at any time t is a function of time. In this functional dependence, however, the velocity's x-component V_x remains constant while its y-component V_y changes with time.

When the initial position is taken to be $x_0 = 0$ in Eq. 2.17, the horizontal distance to the object's landing position (x) is obtained as

$$x = V_{0x}t = V_0t \tag{2.23}$$

where t is the time of which the object stays in the air, or it is time of flight.

In Eq. 2.18, if we substitute $y_0 = 0$ and $V_{oy} = 0$, the vertical distance (y) taken by the object is

$$y = -\frac{1}{2}gt^2 \tag{2.24}$$

The time of flight for the object is defined by the vertical displacement y. The time t can be obtained from Eq. 2.24 as

$$t = \sqrt{\frac{2y}{g}} \tag{2.25}$$

As seen in Figure 2.3, time of flight (t) for an object launched at a certain height of $h = -y$ can be calculated by Eq. 2.25. In addition, using measured horizontal distance (x) and calculated t values, according to Eq. 2.23, the object's initial velocity V_0 can be found as

$$V_0 = V_{0x} = \frac{x}{t}. \tag{2.26}$$

2.2.2. Projectile Motion

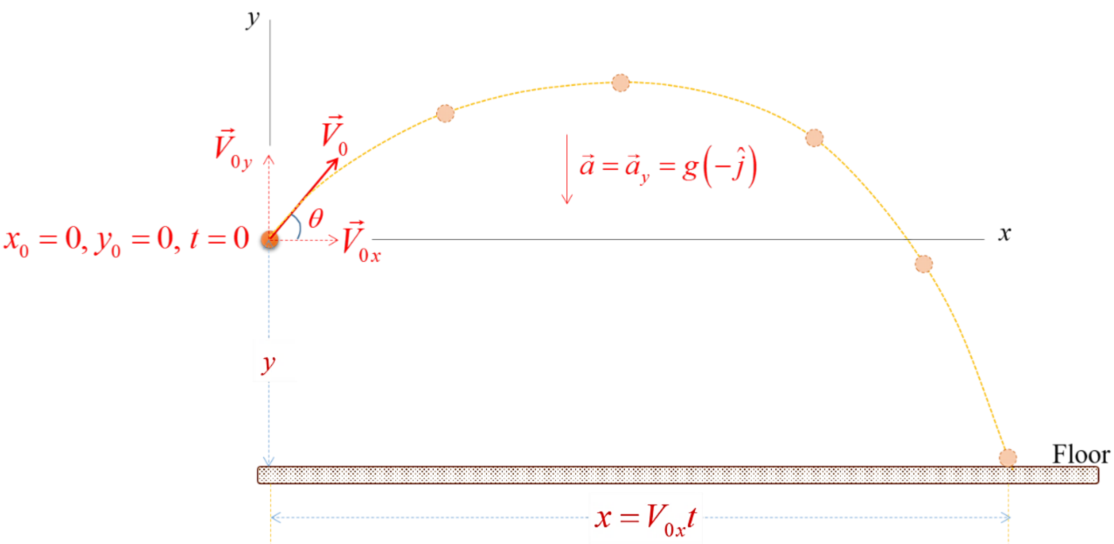


Figure 2.4. Projectile motion.

The motion of an object launched from an initial position at an initial angle θ from the horizontal under the influence of Earth's gravitational acceleration is called "***Projectile Motion***". As seen in Figure 2.4, let's consider the case when the object is launched from an initial position $x_0 = 0$ and $y_0 = 0$ with an initial angle θ and initial velocity $\vec{V}_0$. If the air resistance is neglected, the object's acceleration vector can be defined as

$$\vec{a} = a_x\hat{i} + a_y\hat{j} = -g\,\hat{j} \tag{2.27}$$

$$a_x = 0$$

$$a_y = g = \text{constant}$$

In projectile motion, we observe a uniform linear motion in *x*-direction and a motion with constant acceleration in *y*-direction.

The components of the object's initial velocity $\vec{V}_0$ can be written as

$$V_{0x} = V_0 \cos\theta \tag{2.28}$$

$$V_{0y} = V_0 \sin\theta. \tag{2.29}$$

The vertical distance taken by the object in the projectile motion is given by

$$y = y_0 + V_{0y}t - \frac{1}{2}gt^2 \tag{2.30}$$

Where y_0 is the initial vertical position of the object, and *y* is the vertical displacement of the object. When we substitute Eq. 2.29 into Eq. 2.30 and when we consider the object is launched from $x_0 = 0$ and $y_0 = 0$ position at *t*=0 (see Figure 2.4), the vertical distance is obtained as

$$y = (V_0 \sin\theta)t - \frac{1}{2}gt^2 \tag{2.31}$$

where *t* is the time of flight. If the object's initial velocity V_0 and initial angle θ are known, the object's time of flight can be calculated by using Eq. 2.31.

The object's horizontal displacement when it hits the floor is given by

$$x = V_{0x}t. \tag{2.32}$$

When we substitute Eq. 2.32 into Eq. 2.28, we obtain

$$x = (V_0 \cos\theta)t \tag{2.33}$$

If the object's initial velocity V_0 and initial angle θ are known and the object's time of flight is calculated, the object's horizontal displacement can be found by using Eq. 2.33.

2.2.3. Horizontal Range

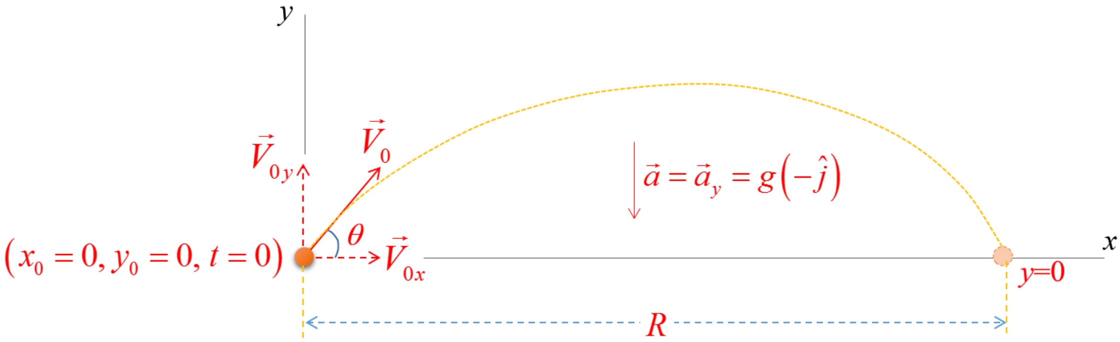


Figure 2.5. Horizontal range in the projectile motion.

The horizontal distance between the positions where the object is launched and then is landed is called "***Range, R***". Range is given by

$$R = V_{0x} t. \tag{2.34}$$

As seen in Eq. 2.5, the object launched with an initial velocity $\vec{V}_0$ and initial angle θ from the origin takes a definite horizontal distance (R), after that, it reaches to the same height, i.e., $y = 0$. Using Eq. 2.30, time of flight is obtained as

$$t = \frac{2V_{0y}}{g} = \frac{2V_0 \sin\theta}{g} \tag{2.35}$$

By substituting Eq. 2.35 into Eq. 2.34 and making use of trigonometric relations, the range equation becomes

$$R = \frac{V_0^2 \sin 2\theta}{g}. \tag{2.36}$$

This expression is only valid for the case such that the launch and landing heights are equal.

PROCEDURE:

A. Ballistic Pendulum

Given Data:

$R_{CM}=20.7$ cm (Distance from the pivot point to the center of mass)

$d=1.585$ cm (Diameter of the steel ball)

$m=16$ g (Mass of the steel ball)

$m_p=17$ g (Mass of the pivot)

$m_ç=24$ g (Mass of the rod)

$m_b=20$ g (Mass of the block)

$M= m_p+ m_ç+ m_b=61$ g (Total mass of the pendulum)

A1. Insert the steel ball into the launcher, and compress the spring up to the first position (short range).

A2. Make sure the angle indicator reads zero degree ($\theta = 0°$) when the pendulum is hanging in the vertical position (i.e., at the initial equilibrium position).

A3. Shoot the ball into the pendulum by pressing the button on the launcher when the pendulum is at rest. Notice that the ball should remain within the catcher as the pendulum swings to its maximum (highest) position.

A4. Read the maximum angle (θ) value recorded at the angle indicator and record it in Table 2.1.

A5. Use Eq. 2.9 to calculate the maximum vertical displacement of the center of mass and record it in Table 2.1.

A6. Use Eq. 2.7 to calculate the magnitude of the velocity of the pendulum-ball system right after the collision (take $g = 9.8\ \text{ms}^{-2}$) and record it in Table 2.2.

A7. Use Eq. 2.8 to calculate the magnitude of the initial horizontal velocity of the ball right after it leaves the launcher and record it in Table 2.2.

A8. Use Eq 2.3 to calculate the total kinetic energy of the ball before the collision and record it in Table 2.2.

A9. Use $K' = \frac{1}{2}(m+M)V'^2$ to calculate the total kinetic energy of the pendulum-ball system right after the collision and record it in Table 2.2. Interpret your results.

A10. Calculate the total mechanical energy E and E' of the system before and after the collision and record it in Table 2.2. Interpret your results.

A11. Use $P = mV$ to calculate the magnitude of the momentum of the ball before the collision and record it in Table 2.2.

A12. Use $P' = (m + M)V'$ to calculate the magnitude of the momentum of the pendulum-ball system right after the collision and record it in Table 2.2. Interpret your results.

A13. Repeat the experiment for the spring's second position setting (long range).

B. Horizontal Projectile Motion

B1. Mount the launcher on the table and align the front of the launcher with one edge of the table as seen in Figure 2.6. Place the ball into the launcher and set the mechanism to the first position setting.

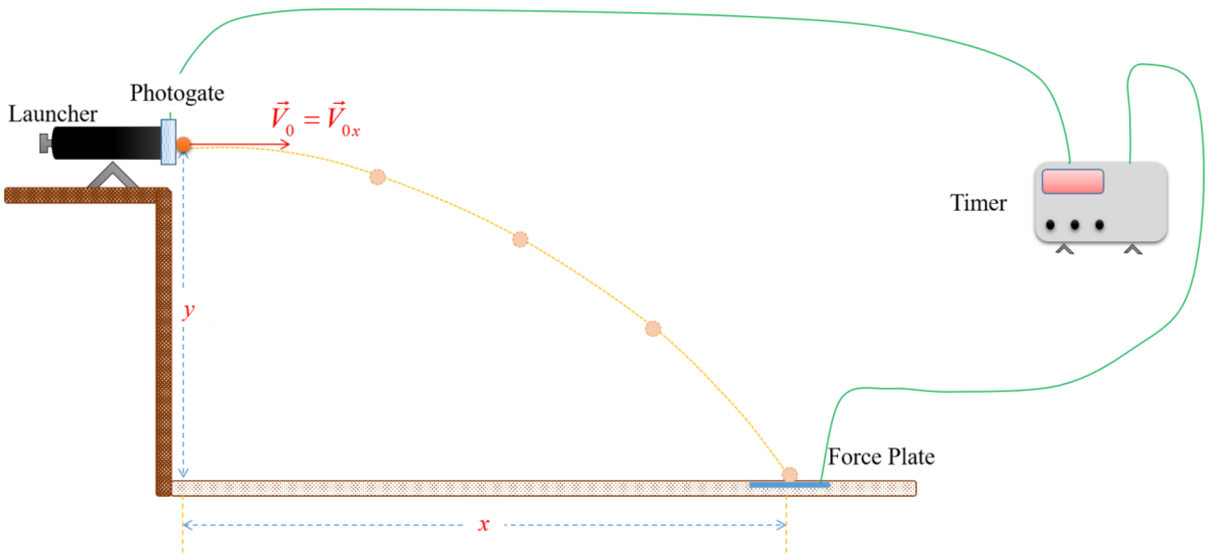


Figure 2.6. Experimental setup for horizontal projectile motion.

B2. Adjust the angle of the launcher to $\theta = 0°$ degrees.

B3. Before the experiment, you should practice launching the ball to decide where to place the "*time of flight plate (force plate)*" on the floor for the time measurement. Then, place the plate at the location where the ball lands, and place first a piece of white paper then a piece of carbon copying paper on top of the plate. So, the ball leaves a mark on the paper when it hits the floor.

B4. After re-setting the launching mechanism, place the photogate on the exit of the barrel and make the connections to the timer system using the cables.

B5. Hit the "*Start*" button on the timer, and launch the ball.

B6. Read the time t_1 on the timer and record it in Table 2.3. That is the time interval taken by the ball for a distance of the ball's diameter (d).

B7. Read the time t_2 on the timer and record it in Table 2.3. That is the time of flight (t_{tof}) for the ball.

B8. Use $v_0 = \dfrac{d}{t_1}$ to calculate the magnitude of the initial velocity of the ball as it leaves the barrel and record it in Table 2.3.

B9. Measure the vertical distance from the bottom of the barrel of the launcher to the floor (y) and the horizontal distance to the floor (x), and record them in Table 2.3 (See Figure 2.6).

B10. Use $t'_{tof} = \sqrt{\dfrac{2y}{g}}$ to theoretically calculate the time of flight for the ball and record it in Table 2.3. Compare the calculated value with the measured value from the timer and interpret the results.

B11. Use $x = V'_0\, t_{tof}$ to calculate the initial velocity (V'_0) of the ball as it leaves the barrel and record it in Table 2.3. Compare this velocity with the velocity (V_0) calculated by using the diameter of the ball and the time t_1 and interpret the results.

B12. Repeat the experiment for the spring's second position setting (long range).

C. Projectile Motion

C1. Construct the experimental setup shown in Figure 2.7 and adjust the angle of the launcher as $\theta = 10°$.

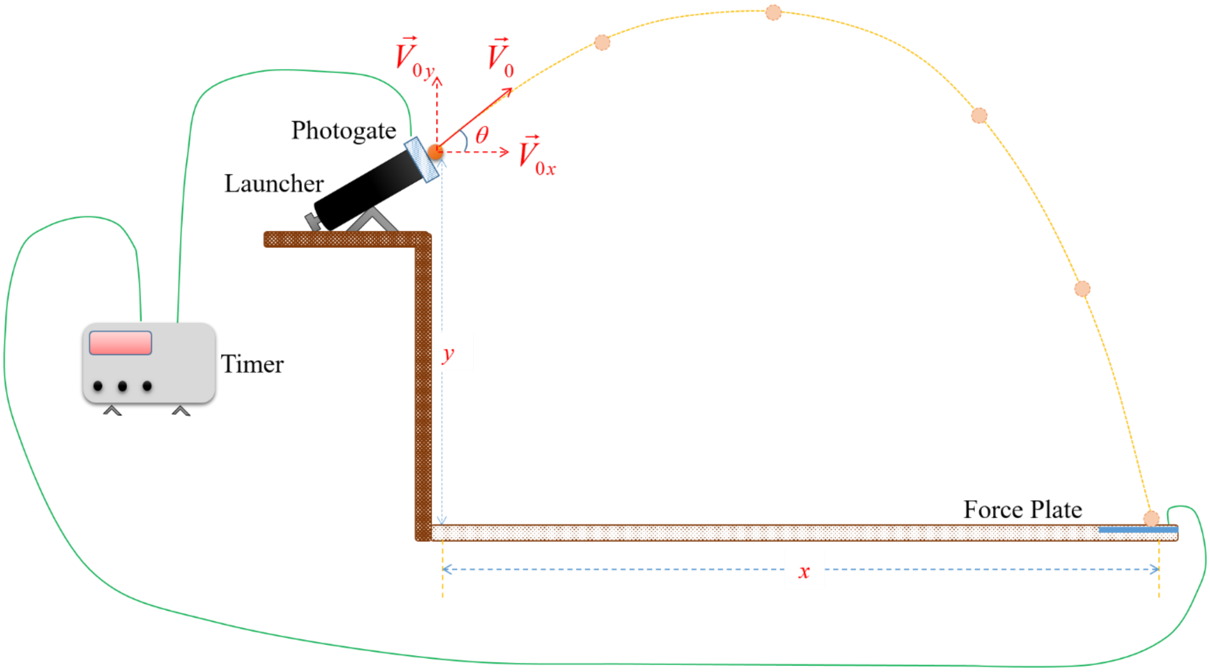


Figure 2.7. Experimental setup for the projectile motion.

C2. After the practice shots, place the photogate and time of flight plate properly in place.

C3. When the launcher is at the first position, launch the ball, read times t_1 and t_2 on the timer and record them in Table 2.4.

C4. Use $V_0 = \dfrac{d}{t_1}$ to calculate the magnitude of the initial velocity of the ball as it leaves the barrel and record it in Table 2.4.

C5. Use $V_{0x} = V_0 \cos\theta$ and $V_{oy} = V_0 \sin\theta$ to calculate the velocity components and record the results in Table 2.4.

C6. Measure the horizontal distance x from the projection of the release point of the launcher on the table to the point where the ball hits on the floor and record it in Table 2.4.

C7. Use $x' = V_{0x} t_{tof}$ to theoretically calculate the horizontal distance taken by the ball and record it in Table 2.4.

C8. Compare the calculated x' value with the measured x value and interpret your results.

C9. Repeat the experiment for $\theta = 20°$ and $\theta = 30°$ settings.

EXPERIMENTAL DATA:

TABLE 2.1

m (kg)	M (kg)	R_{CM} (m)	Range	θ (°)	h (m)
			Short		
			Long		

TABLE 2.2

Range	V (ms^{-1})	V' (ms^{-1})	K (J)	K' (J)	E (J)	E' (J)	P ($kgms^{-1}$)	P' ($kgms^{-1}$)
Short								
Long								

TABLE 2.3

θ (°)	d (m)	Range	t_1 (s)	$t_2 = t_{tof}$ (s)	V_0 (ms^{-1})	y (m)	x (m)	t'_{tof} (s)	V'_0 (ms^{-1})
		Short							
		Long							

TABLE 2.4

θ (°)	t_1 (s)	$t_2 = t_{tof}$ (s)	V_0 (ms^{-1})	V_{0x} (ms^{-1})	V_{0y} (ms^{-1})	x (m)	x' (m)
10							
20							
30							

FREE FALL AND ATWOOD'S MACHINE

M3

FREE FALL AND ATWOOD'S MACHINE

M3

PURPOSE OF THE EXPERIMENT:

1. To observe the gravitational force that enables objects to move towards the center of the Earth.

2. To investigate the motion with acceleration for objects by the effect of gravity.

3. To calculate the acceleration of gravity.

4. To investigate the motion with constant acceleration by the Atwood's machine.

5. To apply Newton's second law of motion.

EQUIPMENT FOR THE EXPERIMENT:

Three balls with different masses

Ball holder

Photogate

Stopwatch

Atwood's machine

THEORY:

3.1. FREE FALL

As seen in Figure 3.1, when an object at a certain height is dropped from rest, the object increases its speed, and eventually hits the ground. This motion of the object is called "***free fall***". Free fall motion is caused by a force that attracts objects toward the center of the Earth. This force is called as the "***gravitational force***". When an object is released from rest, neglecting air resistance, it begins to move toward the ground under the influence of its weight with constant gravitational acceleration. The magnitude of the gravitational acceleration is $g = 9.8\ \mathrm{ms^{-2}}$ and it is always directed toward the Earth's center.

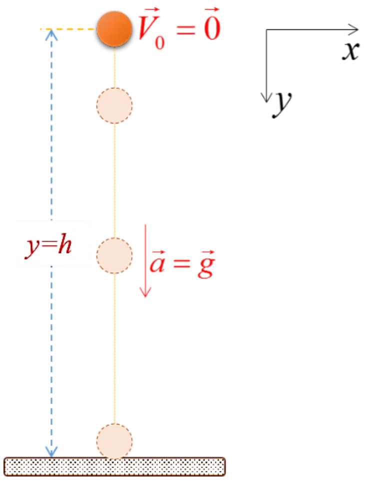


Figure 3.1. Free fall motion.

In the case of the height at which the object falls is not too large, and the air resistance is negligible; the magnitude of the gravitational acceleration constant, g, can be taken constant. Therefore, the free fall motion can be considered as "***one-dimensional motion with constant acceleration***". The formulas of a motion with constant acceleration can be rewritten by considering the free fall motion which takes place along vertical (y) direction, and by using $\vec{g}$ instead of $\vec{a}$ for constant acceleration. The sign of the gravitational acceleration, $\vec{g}$, should be taken consistently by the positive y-axis selection. If the positive y-axis is selected in the upward direction; $\vec{a} = -\vec{g}$, if the positive y-axis is selected in the downward direction; $\vec{a} = \vec{g}$.

As seen in Figure 3.1, if an object of mass m is released from rest at height $y=h$ at the time $t_1 = 0$, the object takes a vertical distance y at the time $t_2 = t$. There is a relationship between the gravitational acceleration and the distance y given as

$$\vec{y} = \vec{y}_0 + \vec{V}_0 t + \frac{1}{2}\vec{g}t^2. \tag{3.1}$$

Here, $\vec{y}$ represents the displacement vector at the time t, $\vec{y}_0$ represents the initial vertical position vector, and $\vec{V}_0$ represents the initial velocity vector of the object. According to Figure 3.1, the object's initial position is $\vec{y}_0 = \vec{0}$, and initial velocity is $\vec{V}_0 = \vec{0}$. Hence,

$$\vec{y} = \frac{1}{2}\vec{g}t^2 \tag{3.2}$$

According to Eq. 3.2, if the time taken for the object to reach the falling distance y is measured, the magnitude of the gravitational acceleration can be calculated by

$$g = \frac{2y}{t^2}. \qquad (3.3)$$

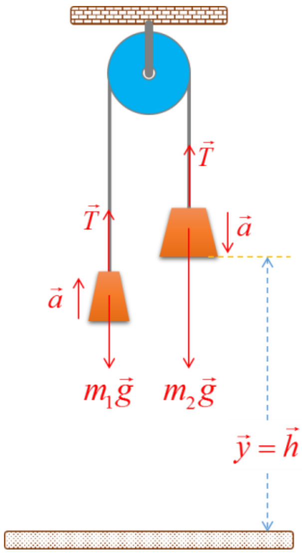
In addition, if $y = f\left(t^2\right)$ graph is plotted, a straight line with a constant slope *m* passing through the origin is obtained:

$$m = \tan\theta = \frac{1}{2}g. \qquad (3.4)$$

So, the magnitude of the gravitational acceleration, *g*, can be found experimentally from Eq. 3.4. In free fall motion, $y = f(t)$ graph is a parabola.

3.2. ATWOOD'S MACHINE

An Atwood's machine is a device used to determine the acceleration of a system due to gravity. With this device, the linear motion of the system with constant acceleration is investigated. The Atwood's machine seen in Figure 3.2 consists of two different masses $(m_2 > m_1)$ connected by a nonflexible string with negligible mass that passes over a frictionless pulley.



Şekil 3.2. Atwood's machine and forces acting on both masses. The string and the pulley are considered massless at Atwood's machine, and the pulley has the ability to rotate freely about its axis.

To move an object which is initially stationary, a force must be applied on that object. The forces acting on the hanging objects with masses m_1 and m_2 in Atwood's machine are the tension force in the string $(\vec{T})$ and the weights of the objects ($m_1\vec{g}$ and $m_2\vec{g}$), as seen in Figure 3.2. The mass difference between two hanging objects $(m_2 - m_1)$ determines the net force acting on the system. This net force accelerates both of the hanging masses. When both masses of the system is released from rest, both masses move with constant acceleration. In this case, the heavier mass, m_2, moves downward with constant acceleration, the lighter mass, m_1, moves upward with the same acceleration.

The resultant force acting on an object is called as "***Net Force***". If there is a net force acting on an object, the object accelerates in the direction of the net force. This is known as "***Newton's II. Law of Motion***".

$$\vec{F}_{net} = m\vec{a} \tag{3.5}$$

$\vec{F}_{net}$: The net force acting on the object

m : Mass of the object

$\vec{a}$: Acceleration of the object

According to Newton's II. Law of Motion, acceleration of an object is proportional to the net force acting on the object and inversely proportional to the mass of the object. For example; if the net force is doubled, the acceleration increases by two-fold.

$$\vec{a} = \frac{\vec{F}_{net}}{m} \cdot \tag{3.6}$$

The unit of force is given by newton (N) in SI system. 1 newton is defined as the amount of force which is required to accelerate a 1-kg mass with 1 ms^{-2} acceleration (1 N = 1 kgms^{-2}).

Since there are net forces acting on hanging objects of mass m_1 and m_2 in the Atwood mechanism, Newton's II. Law of Motion can be applied independently to these objects. In the mechanism, the objects are not in equilibrium and they pick up speed. Eq. 3.5 can be written for both objects, and the equations of motion are given by

$$F_{net1} = T - m_1 g = m_1 a \tag{3.7}$$

$$F_{net2} = m_2 g - T = m_2 a. \tag{3.8}$$

Combining Eq. 3.7 and Eq. 3.8 side by side and cancelling out the tension force *T*, the acceleration of the system is obtained as

$$a = g\left(\frac{m_2 - m_1}{m_1 + m_2}\right) \cdot \tag{3.9}$$

This gives the theoretical acceleration value for the system that consists of the masses m_1 and m_2. As seen in Eq. 3.9, the value of the acceleration that is derived from Newton's II. Law of Motion is equal to gravitational acceleration g multiplied by the mass difference divided by the total mass. Consequently, the net force on the mass system in Atwood's machine is equal to the gravitational acceleration times the mass difference, $g\left(m_2 - m_1\right)$. In other words, the mass difference $\left(m_2 - m_1\right)$ determines the net force acting on these two objects, and this force accelerates the hanging masses. Thus, while the heavier mass $\left(m_2\right)$ moves downward with constant acceleration, the lighter mass $\left(m_1\right)$ accelerates upward. If $m_1 = m_2$, regardless of the location of the masses, the system is in equilibrium.

As seen in Figure 3.2, if the mass m_2 is released vertically from rest $\left(\vec{V}_0 = 0\right)$ at a height $y = h$ with respect to the ground at the time $t_1 = 0$, its displacement vector becomes $\vec{y}$. The magnitude of the displacement vector is given by

$$y = y_0 + v_0 t + \frac{1}{2}at^2. \tag{3.10}$$

In Atwood's machine, when the initial position of mass m_2 is taken $y_0 = 0$ and initial velocity is taken $V_0 = 0$, the displacement of the object is obtained as

$$y = \frac{1}{2}at^2 \tag{3.11}$$

The displacement (y) (i.e., the distance taken) is proportional to the square of the time. If the distance (y) and the time (t) taken for the mass m_2 to reach that distance are both measured, the acceleration of the system can be calculated by using Eq. 3.11. As a result, the magnitude of the acceleration, in terms of the measured quantities displacement (y) and time (t), is given by

$$a = \frac{2y}{t^2} \cdot \tag{3.12}$$

This value gives the experimental acceleration. If $y = f\left(t^2\right)$ graph is drawn, a straight line passing through the origin is obtained, and the slope of this line gives the magnitude of the acceleration of the system:

$$m = \tan\theta = \frac{1}{2}a. \tag{3.13}$$

The magnitude of the system's acceleration can be found experimentally from the slope of the $y = f\left(t^2\right)$ graph obtained by using the measured falling times for different vertical y distances data of mass m_2.

PROCEDURE:

A. Free Fall

A1. Set the distance between the plates (y) as 25 cm by moving the photogates in Figure 3.3.

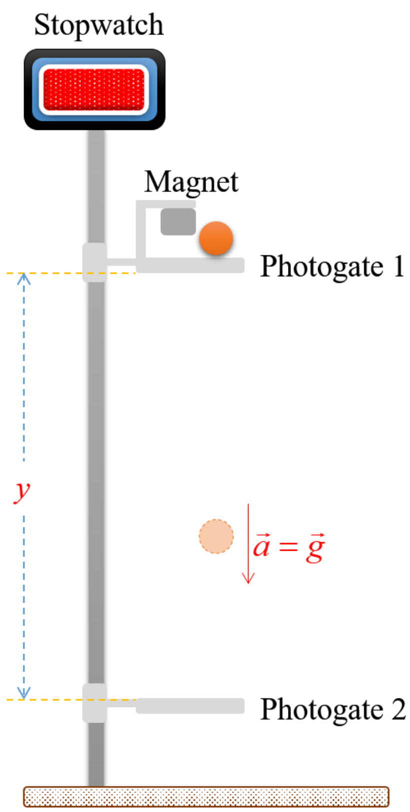


Figure 3.3. Experimental setup for the free fall motion.

A2. Place the ball that you use to the holder. Next, press the magnet control button to start the motion.

A3. Read the elapsed time, for the ball to take the distance y, from the stopwatch in the set-up, and then record the result in Table 3.1.

A4. Repeat these steps two more times, read t_2 and t_3 times and record them in Table 3.1.

A5. Calculate the arithmetic average (t_{ave}) of times t_1, t_2, t_3 and t_{ave}^2 values, and then record the results in Table 3.1.

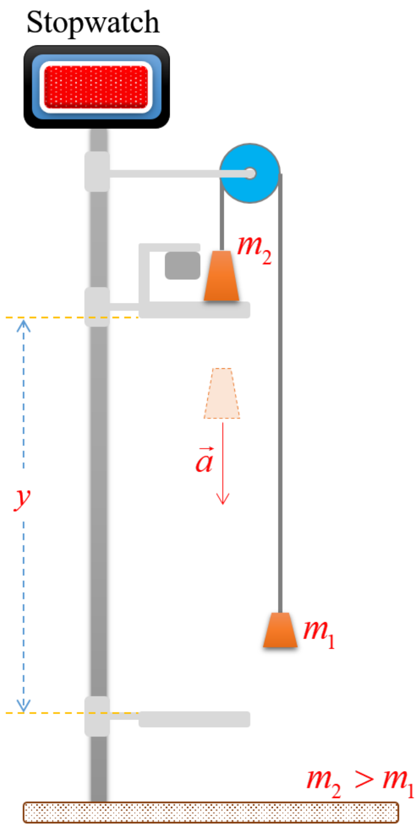
A6. Calculate the acceleration using $a = 2y/t_{ave}^2$ and record the result in Table 3.1.

A7. Repeat all of the steps for 50 cm and 75 cm distances and record the results in Table 3.1.

A8. Draw $y = f\left(t^2\right)$ graph using Table 3.1. Use the slope of this graph to calculate the magnitude of the acceleration of the ball's motion and record the result in Table 3.1. Interpret your results.

B. Atwood's Machine

B1. Design the experimental setup as seen in Figure 3.4.



Şekil 3.4. Experimental setup for the Atwood's machine.

B2. Measure the masses and record the lighter mass (m_1) and heavier mass (m_2) in Table 3.2.

B3. Connect the masses m_1 and m_2 together using an inflexible string with negligible mass, and hang the string around the frictionless fixed pulley. In this case, the top of the heavier mass (m_2) should be attached to the magnet on Photogate 1.

B4. Moving the photogates at the setup, set the distance between the plates (y) as 40 cm.

B5. After completing your checks, press the magnet control button to release (V_0=0) the mass m_2 from its initial position (y=0). During this time, according to Newton's II. Law of Motion, the mass will take the distance (y=40 cm) between photogates with a constant acceleration.

B6. Record the duration of the fall (t_1) that appears in the display in Table 3.2.

B7. Repeat the steps 5 and 6 two more times to minimize the error, read the times t_2 and t_3, and record them in Table 3.2.

B8. Calculate the arithmetic average (t_{ave}) of the times t_1, t_2, t_3 and t_{ave}^2 values obtained for the same height, and then record the results in Table 3.2.

B9. Calculate the magnitude of the constant acceleration of motion using $a = 2y/t_{ave}^2$ and record the result in Table 3.2.

B10. Use Eq. 3.9 to calculate the value for the theoretical acceleration, a', and record it in Table 3.2.

B11. Compare the experimental and the theoretical acceleration values, and interpret your results.

B12. Repeat all of the steps of the experiment for $y = 80\,\text{cm}$.

EXPERIMENTAL DATA:

TABLE 3.1

y (cm)	t_1 (s)	t_2 (s)	t_3 (s)	t_{ave} (s)	t_{ave}^2 (s^2)	a (ms^{-2})	a' (ms^{-2})
25							
50							
75							

TABLE 3.2

y (cm)	m_1 (kg)	m_2 (kg)	t_1 (s)	t_2 (s)	t_3 (s)	t_{ave} (s)	t_{ave}^2 (s^2)	a (ms^{-2})	a' (ms^{-2})
40									
80									

SIMPLE PENDULUM AND CONSERVATION OF ENERGY

M4

SIMPLE PENDULUM AND CONSERVATION of ENERGY

M4

PURPOSE OF THE EXPERIMENT:

1. To determine magnitude of the acceleration of gravity using a simple pendulum.
2. To investigate the period, frequency, and angular velocity of a simple pendulum.
3. To determine the transition velocity of an oscillating object at its equilibrium position.
4. To learn the conservation of energy by experimental investigation of the transformation between potential and kinetic energies.

EQUIPMENT FOR THE EXPERIMENT:

Pendulum system

Masses

Photogate

THEORY:

4.1. SIMPLE HARMONIC MOTION

A simple pendulum is a mechanical system which oscillates periodically. It consists of a mass m suspended at the end of a holder (its weight is neglected) of length L. If the object has a negligible size and the string or rod is massless, then the pendulum is called a simple pendulum. The upper end of the pendulum holder is stable, thus harmonic motion of oscillating mass occurs in a plane. A motion that repeats itself with equal time intervals at near the equilibrium position with the effect of restoring force is called "***oscillation motion***" or "***simple harmonic motion***". Harmonic motion of a simple pendulum is given in Figure 4.1.

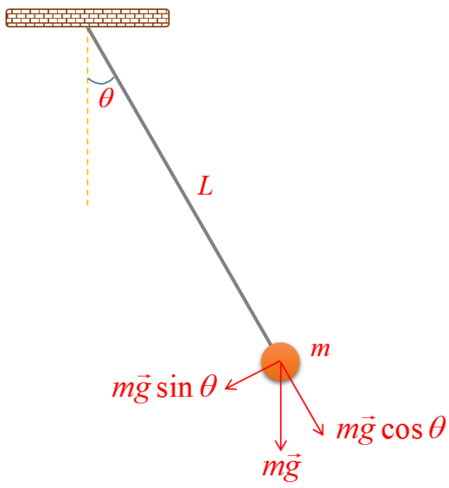


Figure 4.1. Harmonic motion of a simple pendulum.

The time taken by an object to complete one oscillation is called as "***period*** "and is represented by T. The "***frequency***" is defined as the number of cycles per unit time and is represented by f. The relationship with the period and frequency is given by

$$T = \frac{1}{f}. \tag{4.1}$$

The unit of period is second (s) and the unit of frequency is s^{-1} or Hertz. The angular velocity (ω) of a pendulum is the angular displacement per unit time and is defined in terms of frequency or period;

$$\omega = 2\pi f = \frac{2\pi}{T}. \tag{4.2}$$

The unit of angular velocity is $rads^{-1}$.

In a simple pendulum system, a restoring force causes the harmonic motion of an object of mass m. As seen in Figure 4.1 when an object of mass m in a pendulum system is displaced an angle θ from its equilibrium position ($\theta = 0°$), there will be a restoring force that moves the pendulum back towards its equilibrium position. The restoring force is the x-component of the net force (F_x) acting on the object of mass m oscillating in two dimensional plane. We can understand from deflection of the object of mass m at equilibrium position whether this force is a restoring force. That is, the net force acting on the body is always directed towards the equilibrium position as the pendulum swings.

As seen in Figure 4.1, the force acting on the object of mass m is its weight. The x-component of this force (F_x) is the restoring force:

$$F_x = -mg\sin\theta, \tag{4.3}$$

where the minus (-) sign indicates that the force is a restoring force. For small angles $(\theta \leq 10°)$, we can use approximation of $\sin\theta \sim \theta$. Hence, Eq. 4.3 can be written as

$$F_x = -mg\theta. \tag{4.4}$$

Assume an object is suspended by a string of length L and is displaced from its equilibrium position by an angle θ as shown in Figure 4.1. The displacement x of the object is given by

$$x = L\sin\theta \approx L\theta. \tag{4.5}$$

Thus, the angular displacement is given by;

$$\theta = \frac{x}{L}\cdot \tag{4.6}$$

Substituting Eq. 4.6 into Eq. 4.4, the force F_x is obtained as;

$$F_x = -\frac{mg}{L}x = -kx \tag{4.7}$$

where k is the force constant and given by

$$k = \frac{mg}{L}\cdot \tag{4.8}$$

As seen clearly in Eq. 4.7, in this special case of small angles the restoring force (F_x) is proportional to the displacement x of the object in a simple harmonic motion. On the other hand, in the case of higher angles, the restoring force (F_x) in the pendulum system will not be proportional to the displacement x. Therefore, these kind of motions are not considered as simple harmonic motion.

The angular velocity (ω) related to the mass of the body and the force constant is expressed as;

$$\omega = \sqrt{\frac{k}{m}}\cdot \tag{4.9}$$

If Eq. 4.8 is substituted into Eq. 4.9, the angular velocity of the motion is obtained as;

$$\omega = \sqrt{\frac{g}{L}}\cdot \tag{4.10}$$

By using Eq. 4.2 and Eq. 4.10, the frequency of a pendulum (f) is obtained as;

$$f = \frac{\omega}{2\pi} \tag{4.11}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}\cdot \tag{4.12}$$

In this case, the period of a pendulum (T) is;

$$T = \frac{1}{f} = 2\pi\sqrt{\frac{L}{g}}\cdot \tag{4.13}$$

The period of a simple pendulum changes depending on the length of pendulum (L) and gravitational acceleration (g). Also, if the period T and length of pendulum L are known, the value of gravitational acceleration g can be calculated by using Eq. 4.13.

4.2. CONSERVATION OF ENERGY IN A SIMPLE PENDULUM

Mechanical energy of a system is the sum of kinetic and potential energies. If the friction of a system is neglected, the mechanical energy of the system is a constant since energy will not transform into heat. During the motion of an object, potential energy will be transferred to kinetic energy or vice versa. This statement is called as the "***Law of Conservation of Energy***".

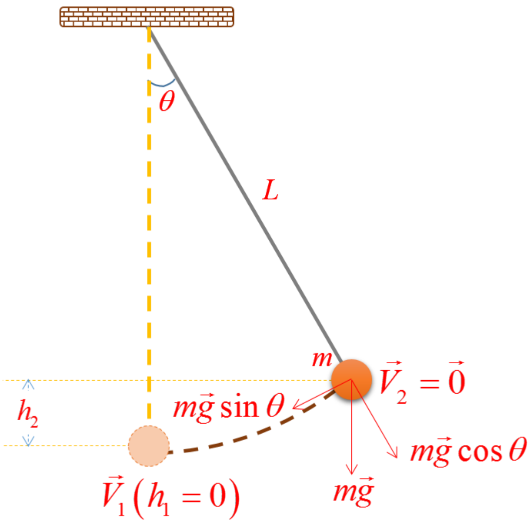


Figure 4.2. Transformation of kinetic energy-potential energy in a simple pendulum.

An object of mass m oscillates as shown in Figure 4.2. Let us consider that the object is released from a height h_2 (initial velocity $V_2 = 0$). In this case, total energy of the object (E) is equal to the potential energy of the object (U) at this point. When the object moves to the point $h_1 = 0$ due to a restoring force, the potential energy of the object will be zero and kinetic energy of the object (K) will be maximum. In other words, total energy of the object is equal to the kinetic energy of the object at this point. When the object again moves to h_2 point, the potential energy of the object will be maximum. Thus, the total energy of the pendulum will stay constant during the oscillations.

According to Figure 4.2, total energy of the object E_i with mass m at height h_2 is equal to the potential energy of the object:

$$E_i = U = mgh_2. \tag{4.14}$$

The potential energy of the pendulum is zero when it passes the equilibrium position $(\theta = 0°$ and $h_1 = 0)$ with a velocity of $\vec{V}_1$ and the total energy (E_f) at this point is equal to the kinetic energy of the oscillating object:

$$E_f = K = \frac{1}{2} m V_1^2. \tag{4.15}$$

According to the conservation of energy, initial (E_i) and final (E_f) mechanical energies must be equal to each other. Thus;

$$mgh_2 = \frac{1}{2} m V_1^2. \tag{4.16}$$

Therefore, for a mechanical system that makes a simple harmonic motion, the transition velocity (V_1) is the velocity when the body of mass *m* passes the equilibrium point and can be calculated experimentally.

PROCEDURE:

A. Determination of the Gravitational Acceleration Experimentally:

Mode 1: It gives the transition time of the pendulum holder's diameter from the equilibrium position (*photogate*).

Mode 2: It gives half period of the simple pendulum (*T/2*).

Mode 3: It gives the period (*T*) for 10 oscillations of the body of mass *m* and transition velocity from the equilibrium position (*V*).

Memory (← →): It represents back and forward values of the period and velocity at the end of 10 periods of oscillations.

A1. Setup the equipment as shown in Figure 4.3.

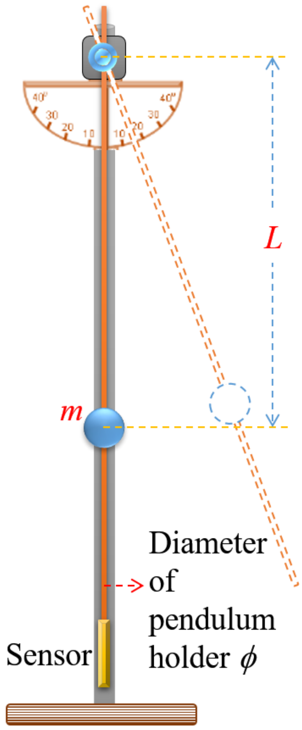


Figure 4.3. Experimental setup of a simple pendulum.

A2. Measure the mass of the object and record it in Table 4.1. Hang this body at the pendulum holder.

A3. Adjust the distance between the point where the pendulum holder is fixed and the center of mass of the object as $L=50$ cm.

A4. Turn on the photogate timer unit (Figure 4.4).

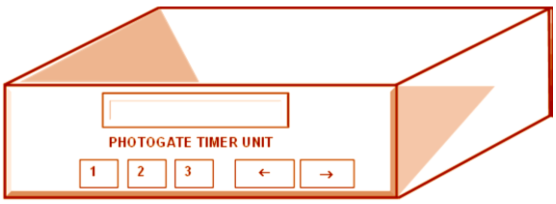


Figure 4.4. Photogate timer unit.

A5. Pull the string aside so that it makes an angle of $\theta = 10°$ from the equilibrium point ($\theta = 0°$) and release the pendulum after pressing the Mode 3 button on the photogate timer unit.

A6. After the pendulum completes 10 oscillations, the period and the transition velocity at the equilibrium point (V) will appear on the timer unit screen, record them in Table 4.1.

A7. Calculate the gravitational acceleration (g) experimentally using Eq. 4.13 and record it in Table 4.1.

A8. Compare the calculated gravitational acceleration (g) and expected value (g') and make comments.

A9. Calculate the angular velocity of the pendulum (ω) by using experimentally calculated period value (T) and record it in Table 4.1.

A10. Find the frequency of the pendulum (f) by using the angular velocity and record it in Table 4.1.

A11. Measure the diameter of the pendulum holder (ϕ) and record it in Table 4.1.

A12. Start an oscillation with a small angle $(\theta = 10°)$ and release the pendulum after pressing the Mode 1 button on the photogate timer unit. Read the recorded data from the screen for passing time (t) of the pendulum holder's diameter (ϕ) through the equilibrium point.

A13. Calculate the pendulum holder's transition velocity through equilibrium point (V') using pendulum holder's diameter (ϕ) and transition time (t). Then compare this velocity with the one on Step 6.

A14. Repeat the measurements for different pendulum lengths (L=60 cm and L=65 cm) at $\theta = 10°$ and record your data in Table 4.1.

A15. Repeat these steps for the angle $\theta = 10°$ and record it in Table 4.2.

B. Conservation of Energy in a Simple Pendulum

B1. Set the distance from the pendulum holder to its fixed point as $L=50$ cm.

B2. Provide an oscillation of the pendulum with an angle $\theta = 10°$. Calculate the height h_2 using $h_2 = R_{CM}\left(1-\cos\theta\right)$ and record on Table 4.3.

B3. Calculate the potential energy of the object at the height h_2 and record it in Table 4.3.

B4. Release the object and simultaneously press the Mode 1 button on the photogate timer unit and calculate the pendulum holder's the velocity (V') when it passes the equilibrium position $\left(h_1 = 0,\ \theta = 0°\right)$ by using the recorded time t_1 and pendulum holder's diameter (ϕ).

B5. Repeat the same process in Step 4 by pushing the Mode3 button and read transition velocity of the object (V) when it passes the equilibrium position. Record the result in Table 4.3. Compare the velocity values V and V'.

B6. Calculate the kinetic energies by using velocity values in Step 5 and record them in Table 4.3.

B7. Make comments on the conservation of energy by comparing the energy values in Steps 3 and 6.

B8. Repeat these steps for different pendulum lengths (L=60 cm and L=65cm) for an angle $\theta = 10°$ and record your results in Table 4.3.

EXPERIMENTAL DATA:

TABLE 4.1											
θ (°)	m (kg)	L (m)	T (s)	V (ms^{-1})	g (ms^{-2})	g' (ms^{-2})	ω ($rads^{-1}$)	f (s^{-1})	ϕ (m)	t (s)	V' (ms^{-1})
10		0.50									
		0.60									
		0.65									

TABLE 4.2											
θ (°)	m (kg)	L (m)	T (s)	V (ms^{-1})	g (ms^{-2})	g' (ms^{-2})	ω ($rads^{-1}$)	f (s^{-1})	ϕ (m)	t (s)	V' (ms^{-1})
5		0.50									
		0.60									
		0.65									

TABLE 4.3								
θ (°)	m (kg)	L (m)	h_2 (m)	U (J)	V' (ms^{-1})	V (ms^{-1})	K (J)	K' (J)
10		0.50						
		0.60						
		0.65						

MOTION ON A FRICTIONAL INCLINED SURFACE

M5

MOTION ON A FRICTIONAL INCLINED SURFACE

M5

PURPOSE OF THE EXPERIMENT:

1. To determine the acceleration of an object that moves on a frictional inclined surface.
2. To obtain static and kinetic friction coefficients.
3. To investigate translational and rotational kinetic energies of a rigid body (disc) that moves in a rolling motion.

EQUIPMENT FOR THE EXPERIMENT:

Assembly of inclined surface

Photogate

LabQuest-2 measurement device

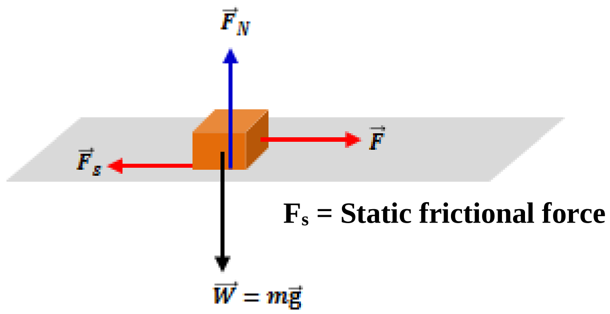
Block and cylinder

THEORY:

5.1. THE FRICTIONAL FORCE

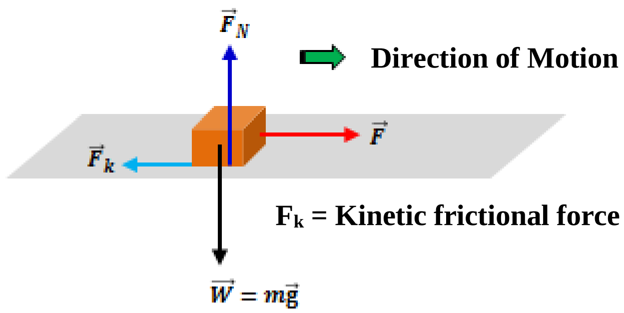
Forces that occur between two surfaces in contact and are opposite to the direction of motion are called as ***'frictional forces'***. The frictional forces can be in two different forms: static and kinetic frictional forces. Therefore, the coefficient of frictional force between an object and surface is different for moving and stationary objects. The frictional force between stationary surfaces is called "***static frictional force,*** $\vec{F}_s$". Applied force $\vec{F}$ parallel to the surface in order to start the motion of an object must be equal and in opposite direction to the static frictional force. Just before the motion of the object, the static frictional force has a maximum value. Once the object starts the motion, the frictional force between the two surfaces reaches to its minimum value. The frictional force between two surfaces in motion and in opposite direction to each other is called "***kinetic frictional force,*** $\vec{F}_k$". The kinetic frictional force is opposite to the direction of motion and smaller than the static frictional force. The static and kinetic frictional forces between an object and surface on a horizontal plane are shown in Figure 5.1. In Figure 5.1 (a), an object of mass m is balanced with its weight $\vec{W}$ and the reaction force $\vec{F}_N$ (normal force) which is applied from the horizontal surface to the object. In order to move the object, a force $\vec{F}$ must be applied on the object. If the force is increased to a certain value, the object starts to move on the surface. To be able to observe the motion, the magnitude of the applied minimum force $\vec{F}$ on the object that is parallel to the plane must be equal to the magnitude of the static frictional force.

Stationary object



(a)

Object in Motion



(b)

Figure 5.1. (a) The static frictional force, **(b)** The kinetic frictional force between an object and surface on a horizontal plane.

The ratio of magnitudes of the static frictional force and the normal force is defined as the "***coefficient of static friction***" and is represented by μ_s. Magnitude of the static frictional force related to the coefficient of static friction is given by

$$F_s = \mu_s F_N . \tag{5.1}$$

After the object starts the motion, magnitude of the kinetic frictional force between the object and the surface is given by

$$F_k = \mu_k F_N \tag{5.2}$$

where, μ_k is the coefficient of kinetic friction. The coefficient of friction has no unit and it depends on the type of frictional surfaces.

5.2. MOTION ON A FRICTIONAL INCLINED SURFACE

As shown in Figure 5.2, when an object is released from an inclined surface which has an angle θ with the horizontal, the object stays stationary up to a certain value of the angle θ. The parallel component of the object's weight to the surface on x-direction, $\vec{F}_x$, and the static frictional force in opposite direction to $\vec{F}_x$ have an effect on a stationary object of mass m on an frictional inclined surface. Furthermore, the perpendicular component of the object's weight on y-direction ($\vec{F}_y$) and the normal force ($\vec{F}_N$) that is applied by the surface have also an effect on the object.

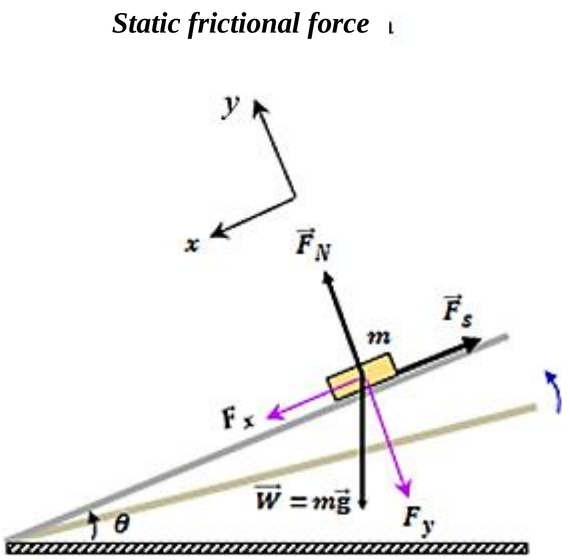


Figure 5.2. Forces on a stationary object on a frictional inclined surface.

In order to determine the coefficient of static friction (μ_s), inclination angle of the inclined surface is increased step by step until the object of mass m reaches to motion limit. At the motion limit (when the object just starts to move), inclination angle of the inclined surface (critical angle, θ_c) is measured. The coefficient of static friction can be calculated from the measured θ_c.

When the object starts to move on a frictional inclined surface at the critical angle value, the net force acts on the system on x-direction is zero;

$$\sum F_x = F_x - F_s = 0$$

$$\sum F_x = F_x - \mu_s F_N = 0$$

$$\sum F_x = mg\sin\theta_s - \mu_s mg\cos\theta_s = ma_x = 0. \tag{5.3}$$

From Eq. (5.3), the coefficient of static friction between the inclined surface and the object is given by

$$\mu_s = \tan\theta_s. \tag{5.4}$$

After the calculation of the coefficient of static friction, magnitude of the static friction force $\left(\vec{F}_S\right)$ can be obtained by using Eq. 5.1.

When the object starts to move, the static frictional force is at its maximum value. After the object starts the motion, the value of the static friction reduces a little bit and then it reaches to a constant value. Due to the motion of the object, a frictional force arises and it is called as **"*kinetic frictional force*"**.

In order to experimentally determine the coefficient of kinetic friction (μ_k), the motion of an accelerating object on an inclined plane is studied. For this reason, inclination angle of the inclined plane (θ) is set to a higher value ($\theta > \theta_C$) of the critical angle (θ_c) to let the object move with a constant acceleration. The motion of released object on a frictional inclined surface with inclination angle θ is shown in Figure 5.3.

Static frictional force

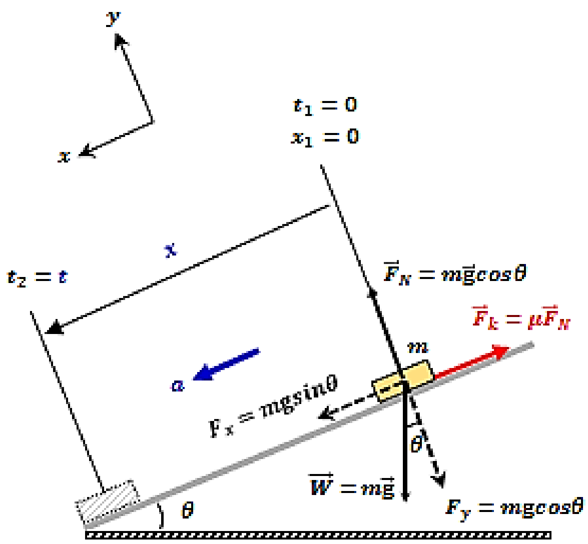


Figure 5.3. A sliding motion on a frictional inclined surface and forces that act on the object.

The magnitude of $\vec{F}_x$ that moves the object on a frictional inclined surface can be calculated by

$$F_x = mg\sin\theta. \tag{5.5}$$

$\vec{F}_y$ is used to determine the magnitude of the frictional force and calculated by

$$F_y = mg\cos\theta. \tag{5.6}$$

Parallel component of the object's weight to the plane ($mg\sin\theta$) forces the object to move downward along the inclined plane. While the perpendicular component of the object's weight ($mg\cos\theta$) pushes the object onto the plane, the plane responds back on the object with a normal force $\vec{F}_N$. When the motion starts, the kinetic friction force which is opposite to the direction of motion arises because of the friction between two surfaces. The acceleration of the object in perpendicular direction to the surface is zero because the object does not move in this direction ($\vec{a}_y = \vec{0}$). Therefore, the magnitude of the net force that acts on the system in *y*-direction can be found by

$$\sum F_y = F_N - F_y = F_N - mg\cos\theta = ma_y = 0. \tag{5.7}$$

According to Eq. (5.7), the magnitude of the normal force $\vec{F}_N$ is given by

$$F_N = mg\cos\theta. \tag{5.8}$$

After the object starts to move, the magnitude of the kinetic frictional force that acts on the object is given by

$$F_k = \mu_k F_N = \mu_k mg\cos\theta. \tag{5.9}$$

The net force that moves the object on the inclined surface is determined by the forces that act on the system in *x*-direction:

$$\sum F_x = mg\sin\theta - F_k = mg\sin\theta - \mu_k mg\cos\theta = ma_x. \tag{5.10}$$

Because the mass *m* has an increasing acceleration on the frictional inclined surface, the magnitude of the kinetic friction force (F_k) is smaller than the component of the object's weight, $F_x = mg\sin\theta$. From Eq. (5.10), the magnitude of object's acceleration is given by

$$a = a_x = g\left(\sin\theta - \mu_k\cos\theta\right) \tag{5.11}$$

and the coefficient of kinetic friction is given by

$$\mu_k = \frac{\left(g\sin\theta - a\right)}{g\cos\theta}. \tag{5.12}$$

The acceleration of the motion in Eq. (5.12) is determined experimentally from

$$a = \frac{2x}{t^2} \tag{5.13}$$

where t the is elapsed time for displacement of the object and is measured experimentally.

5.3. ROLLING MOTION OF A DISC

When a rigid body rolls around its center of mass (CM) with an angular velocity of ω, this motion is called as "***rolling motion***". A body that rolls without slipping on a smooth surface has two types of energies: "***Rotational kinetic energy,*** $\boldsymbol{K_{rot}}$" and "***Translational kinetic energy,*** $\boldsymbol{K_{trans}}$".

Translational kinetic energy arising from the center of mass of a body that moves with a velocity $\vec{V}$ and mass m is given by

$$K_{trans} = \frac{1}{2} m V_{CM}^{2}. \tag{5.14}$$

Rotational kinetic energy of a body that rolls around its center of mass (CM) with an angular velocity of ω depends on the moment of inertia (I_{CM}) and is given by

$$K_{rot} = \frac{1}{2} I_{CM} \omega^{2}. \tag{5.15}$$

A disc of mass M that has a rolling motion on an inclined surface is shown in Figure 5.4. When the disc is released from h_1 height, it moves downward with the effect of gravitational force. Energy of the disc on the top of the inclined surface is equal to its potential energy ($U = Mgh_1$). If the object has a perpendicular displacement of $y = \Delta h$ on the inclined surface at any time t, the reduced potential energy is given by

$$\Delta U = Mg\left(\Delta h\right). \tag{5.16}$$

The loss of potential energy is transferred to the disk as the kinetic energy. Δh depends on the displacement d of the disc on the inclined surface:

$$\Delta h = h_1 - h_2 = d \sin\theta. \tag{5.17}$$

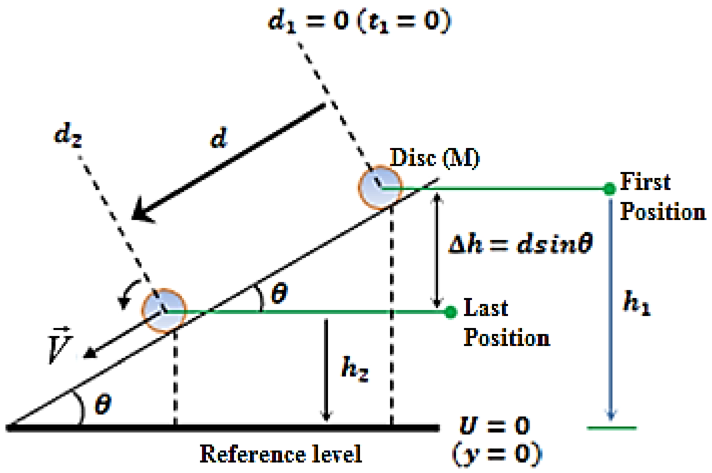


Figure 5.4. Rolling motion of a disc on an inclined surface.

The mechanical energy is conserved unless an external force is applied on the object in a system that is frictionless or has an ignorable coefficient of friction. For the rolling motion of the disc given in Figure 5.4, the initial vertical position, velocity, and angular velocity of the disc are $y = h_1$, $V_i = 0$, and $\omega_i = 0$, respectively. After a given time, the final vertical position, velocity, and angular velocity of the disc are $y = h_2$, $V_f = V$, and $\omega_f = \omega$. According to the conservation of energy, total energies at initial and final positions of the disc must be equal to each other:

$$Mgh_1 = \frac{1}{2}MV^2 + \frac{1}{2}I\omega^2 + Mgh_2 \tag{5.18}$$

$$Mg(h_1 - h_2) = \frac{1}{2}MV^2 + \frac{1}{2}I\omega^2$$

$$Mg(\Delta h) = \frac{1}{2}MV^2 + \frac{1}{2}I\omega^2. \tag{5.19}$$

V is the velocity of the center of mass, I is the moment of inertia about the rotational axis of the disc and $Mg(\Delta h)$ is the change of potential energy of the disc that rolls on the inclined surface.

The moment of inertia for a rolling disc of radius R and mass M about an axis perpendicular to the disc plane and passing through the center of the disc is given by

$$I = \frac{1}{2}MR^2. \tag{5.20}$$

In a rotational motion, the moment of inertia of a rigid body is a time-independent constant and depends on the mass, axis of rotation, and geometric shape of the rigid body.

The relationship between the translational velocity V and the angular velocity ω of a rolling object is given by

$$V = \omega R. \tag{5.21}$$

By substituting Eqs. 5.20 and 5.21 in Eq. 5.19, the velocity of the center of mass of the solid disc is given by

$$V = \sqrt{\frac{4}{3} g\left(\Delta h\right)}. \tag{5.22}$$

If the rolling disc on the inclined surface has a displacement of d, change in its height will be $\Delta h = d \sin\theta$. If Eq. 5.22 is rearranged, velocity of the center of mass becomes:

$$V = \sqrt{\frac{4}{3} g\left(d \sin\theta\right)}. \tag{5.23}$$

The velocity of the center of mass of the rigid object can be experimentally calculated by

$$V = \frac{\phi}{t}, \tag{5.24}$$

where ϕ is called diameter of the disc, which is transition length of the rolling disc through the photogate, and t is the passing time of the disc through the photogate.

According to Eq. 5.21, the angular velocity ω of the rolling disc is given by

$$\omega = \sqrt{\frac{4}{3R^2} g\left(d \sin\theta\right)}. \tag{5.25}$$

On the rolling motion, some of the potential energy lost by the disc of mass M is transferred to the kinetic energy and the disc accelerates on the inclined plane. A part of the potential energy is transferred to the disc as the rotational kinetic energy and the angular velocity ω of disc increases according to Eq. (5.25).

PROCEDURE:

A. Coefficient of Friction and Determination of the Frictional Force

A1. Setup the system as shown in Figure 5.5 (a).

- Connect the photogate 1 and photogate 2 to the inputs of DIG 1 and DIG 2 of the LabQuest-2 measurement device, respectively.
- Set the angle of the inclined surface horizontally ($\theta = 0°$) by using the angle gauge.

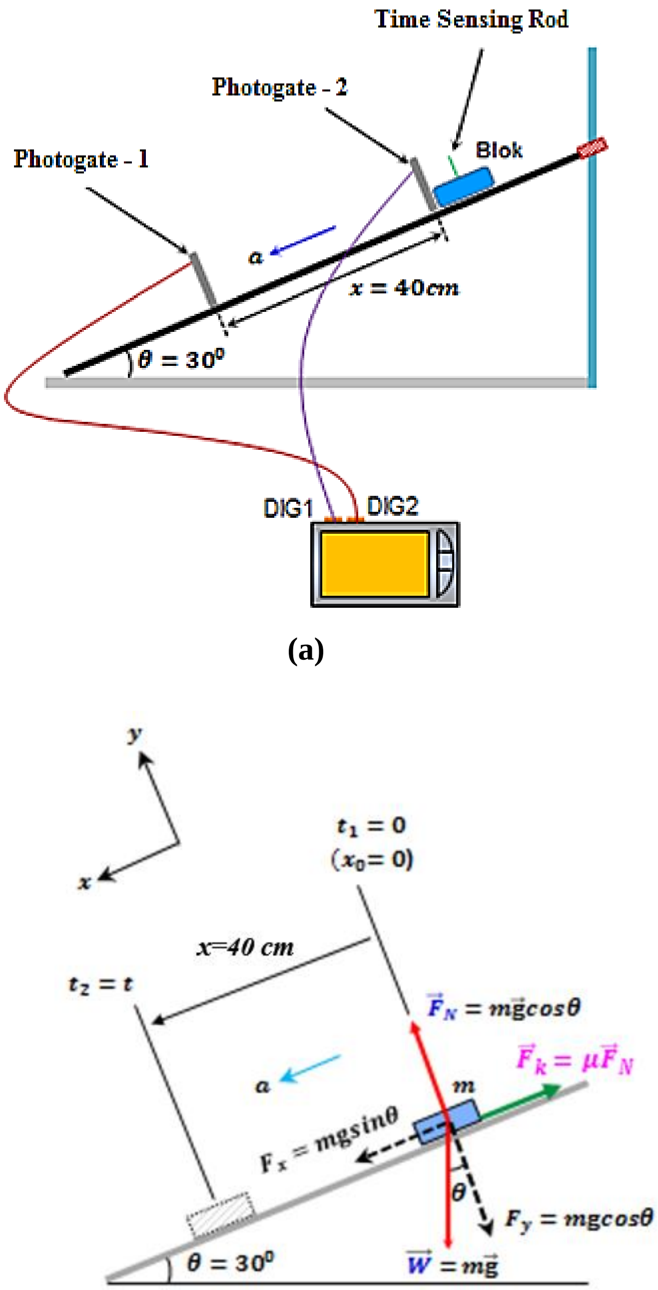


Figure 5.5. (a) Experimental setup for the motion on a frictional inclined surface **(b)** the forces that act on a moving object of mass *m* on an inclined surface.

A2. Determine the critical angle (θ_s) on the frictional inclined surface and record it in Table 5.1.

- Place the block on the inclined surface in horizontal position.
- Increase the angle of the inclined surface gradually until the block starts to move.
- At the time that the block just starts to move on the frictional inclined surface, determine the "***critical angle***".

A3. Use Eq. 5.4 and $F_s = \mu_s mg\cos\theta_s$ to calculate the coefficient of static friction and the static frictional force, then record them in Table 5.1.

A4. Setup the angle of the inclined surface to $\theta = 30°$.

A5. Adjust the distance between photogate 1 and photogate 2 sensors at $x = 40$ cm and record it in Table 5.1. (This distance is the displacement of the released block on the frictional inclined surface).

A6. Turn on the LabQuest-2 measurement device. The following measurement parameters will be displayed on the opening screen:

DIG-1: Gate Mode

Unblocked

DIG-2: Gate Mode

Unblocked

Mode: Photogate Timing

Timing: Motion

A7. Click the "Mode" option on LabQuest-2 opening screen and enter the experimental codes as follows:

Mode: Time Based

Velocity: 100 sample/s

Range: 0.01 s/sample

Time: 10 s

Photogate Mode: Pulse

Distance between the gates: 0.4 m

A8. Click the "Ok" button to return the opening screen. "Mode" option will be changed as "Time Based" on the opening screen. Data collection process of LabQuest-2 device will automatically stop when the measurement time of *10 s*.

A9. Insert the time sensing rod to the input at the top of the block.

A10. Record the total mass of the block as $m = 0.16$ kg in Table 5.2. (Total Mass=Mass of the block+mass of the time sensing rod).

A11. Put the block in front of the photogate 1 on the inclined surface. (There is a friction between the block and the inclined surface)

A12. Click the “Table” symbol to switch from the LabQuest-2 opening screen to the table screen.

A13. To start the data collection process, click the “START” button that is seen on the bottom of the table screen.

A14. Release the block to move between two photogates on the inclined plane. (Data collection process stops automatically when the measurement period is over.)

A15. Click the “Table” button on the LabQuest-2 screen to determine how much time it took for the block to get over a distance of 40 cm on the inclined surface and record it on Table 5.2.

A16. Use Eq. 5.13 to calculate the acceleration of the block that moves on the inclined surface and then record it in Table 5.2.

A17. Use Eq. 5.12 to calculate the coefficient of the kinetic friction and then record it in Table 5.2.

A18. Use Eq. 5.9 to calculate the kinetic frictional force and then record it in Table 5.2.

A19. Repeat the same procedure for $x = 60,\ 80,\ \text{and}\ 100\ \text{cm}$, then calculate the average acceleration and record it in Table 5.2.

A20. Draw the plot of $x = f\left(t^2\right)$ and calculate the average acceleration by using the slope of the graph and record it in Table 5.2. Make comments on the results.

B. Rolling Motion of the Disc

Experimental Data:

Diameter of the disc ϕ=6 cm

Radius of the disc R=3 cm

Mass of the disc M=60 g

Moment of inertia of the disc $I = 27\times10^{-6}$ kgm^2.

B1. Setup the system as shown in Figure 5.6.

- Align the inclined surface horizontally and set the inclination angle at $\theta = 15°$.
- Place the photogate to its holder and then to the lower end of the inclined surface.
- Set the center of mass of the disc, which will have a rolling motion on the inclined surface, and the photogate to be at the same height. In this case transition length of the disc will be equal to the diameter length of the disc (ϕ).

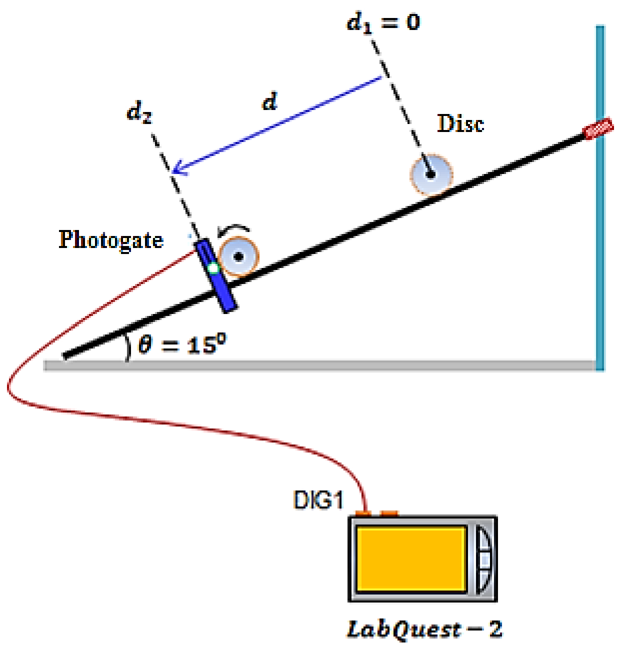


Figure 5.6. Experimental setup for conservation of mechanical energy.

B2. Determine the distance of $d = 30$ cm length from the photogate sensor and mark this point as "Starting Point" for the center of mass of the disc. Therefore, the displacement of the center of mass of the disc, which makes a rolling motion, will be $d = 30$ cm.

B3. Run the LabQuest-2 measurement device and click on the "Mode" button. From the screen, set the parameters as follows:

Mode: Photogate Timing

Photogate Mode: Gate

Length of Object: 0.06 m

then, select the following.

End data collection: With stop button.

Then, choose the "Ok" button to return the opening screen.

B4. Place the center of mass of the disc at pre-marked "Starting Point" on the inclined surface and then keep it fixed.

B5. Press "Table" symbol on LabQuest-2 measurement device. Press the "Start" button that can be seen from the Table screen and then set the disc free.

B6. Press the "Stop" button when the disc passes through the photogate. The disc would be covered the distance of $d = 30$ cm along the rail in the inclined surface.

B7. Read the passing time for disc from the LabQuest-2 "Table" screen and record it in Table 5.3.

B8. When the disc moves with a distance of 30 cm on the inclined surface, calculate the change in vertical distance from $\Delta h = d \sin\theta$ and record it in Table 5.3.

B9. Use Eq. 5.24 to calculate the experimental value for the final velocity of the center of mass of the disc and record it in Table 5.3.

B10. Use Eq. 5.22 to calculate the expected velocity value of the center of mass of the disc and record it in Table 5.3.

B11. Use $\omega = \dfrac{V}{R}$ to calculate the angular velocity of the disc and record it in Table 5.3.

B12. Use $K_{trans} = \dfrac{1}{2}MV^2$ and $K_{rot} = \dfrac{1}{2}I\omega^2$ to calculate the translational kinetic energy and the rotational kinetic energy for the center of mass of the disc and record them in Table 5.3.

B13. Find the total kinetic energy K_{tot} of the rolling disc and record it in Table 5.3.

B14. Use Eq. 5.16 to calculate the decrease in the potential energy of the rolling disc and record it in Table 5.3. Make comments on the conservation of mechanical energy by using the results.

B15. Repeat the same procedure for $\theta = 20°$ and $\theta = 30°$, then make comments on your results.

EXPERIMENTAL DATA:

TABLE 5.1		
θ_s (°)	μ_s	F_s (N)

TABLE 5.2						
					$\theta = 30°$	$m = 0.16$ kg
x (cm)	t (s)	a (ms^{-2})	a_{avg} (ms^{-2})	μ_k	F_k (N)	a_{graph} (ms^{-2})
40						
60						
80						
100						

TABLE 5.3									
θ (°)	t (s)	Δh (m)	V_{exp} (ms^{-1})	V (ms^{-1})	ω ($rads^{-1}$)	K (J)	K_{rot} (J)	K_{tot} (J)	ΔU (J)
15									
20									
30									

SPRINGS, PULLEYS AND PULLEY BLOCKS

M6

SPRINGS, PULLEYS AND PULLEY BLOCKS

M6

PURPOSE OF THE EXPERIMENT:

1. To learn how to find out spring constant using Hooke's Law and oscillation period.

2. To learn how to connect springs in serial and parallel.

3. To examine harmonic motion of a mass hanged on a spring.

4. To understand the importance of fixed-movable pulleys and pulley blocks.

EQUIPMENT FOR THE EXPERIMENT:

Experimental setup of springs and pulleys

Springs with different spring constants

Pulleys (single, double, and triple)

Different masses

Chronometer

Dynamometer

Altimeter

THEORY:

6.1. SPRINGS

As shown in Figure 6.1, when a force $\vec{F}$ is applied on any spring, a tension or compression occurs in the spring. The amount of tension or compression that occurs in the spring (x) depends on the type of the spring and the magnitude of the applied force. For example, a hard spring will compress less than a soft spring under an equivalent applied force.

To compress (or stretch) a spring by a distance of x from the equilibrium position, we need to apply a force which is equal to;

$$\vec{F} = k\vec{x} \tag{6.1}$$

where k, the force constant of the spring (or spring constant), is a measure of the hardness of the spring and it has a different value for each type of springs. Spring constant is always positive, and its unit is $\mathrm{Nm^{-1}}$ in SI.

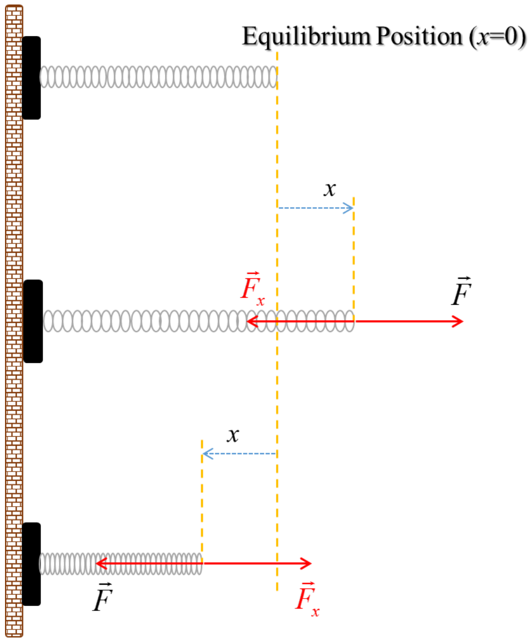


Figure 6.1. Stretched and compressed springs with respect to the equilibrium position by the effect of the force $\vec{F}$.

A spring which moves a distance of x from the equilibrium position due to an applied force $\vec{F}$ will want to return back to its equilibrium position (its normal length). Therefore the spring applies a restoring force ($\vec{F}_x$) in the opposite direction of the force $\vec{F}$.

$$\vec{F}_x = -k\vec{x} \qquad (\textbf{\textit{Hooke's Law}}) \tag{6.2}$$

where (-) sign indicates that it is a restoring force.

Hooke's Law is valid as long as the displacement x is not too large or no permanent deformation occurs on the spring. Furthermore, the restoring force is always towards the equilibrium position.

As seen in Figure 6.2, when a mass m is attached to a stable spring placed in the vertical direction, the spring stretches a distance of x. The spring tension occurs due to the force of gravity acting on the mass (weight). The direction of gravitational force is downwards. When the system is in equilibrium, the weight of the hanging mass is balanced by the spring's restoring force. Thus, the spring will be stretched until the magnitudes of these two forces are equal to each other.

The gravitational force acting on an object with a mass m is given by

$$\vec{F} = \vec{W} = m\vec{g} \tag{6.3}$$

where g is the acceleration of gravity.

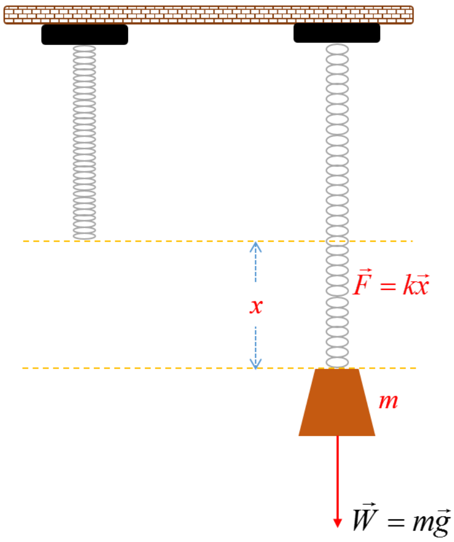


Figure 6.2. A spring in the vertical direction stretches by hanging a body of mass *m*, and its equilibrium position.

On a mass-spring system placed in the vertical direction, forces acting on a body attached to one end of a spring are listed below:

1) The gravitational force acting on the body (*mg*) (downward),

2) The restoring force exerted by the spring (*kx*) (upward).

The spring which is given in Figure 6.2 is stretched to a point where these two oppositely directed forces ($\vec{F}$ and $\vec{W}$) are equal to each other and this point is called "***equilibrium point***". In this case, it can be written as

$$mg = kx. \tag{6.4}$$

According to Eq. 6.4, the mass-spring system is balanced unless any additional external force is applied. If the force has not exceeded the elastic limit, the elongation of the spring is proportional to the force acting on the spring. For example; if a spring is stretched a distance of $x = 10$ cm due to a mass of 1 kg, the same spring will be stretched a distance of $x = 20$ cm with hanging mass of 2 kg. Force-elongation ($F = f(x)$) graphs shows a straight line and its slope gives the spring constant (*k*).

6.1.1. Simple Harmonic Motion

As shown in Figure 6.3, when a body of mass *m* in the equilibrium position is pulled down from *x* = 0 to any position of *x* and then released, the body starts having a "***simple harmonic motion***" between the points *A* and *B*. This motion repeated around the equilibrium position in a certain time interval under the influence of restoring force is called as "***oscillating***

motion ” or “***simple harmonic motion*** ”. The harmonic motion of the mass *m* occurs due to the restoring force of the spring which causes an acceleration towards the equilibrium position. Thus, the mass oscillates back and forth under the influence of the restoring force.

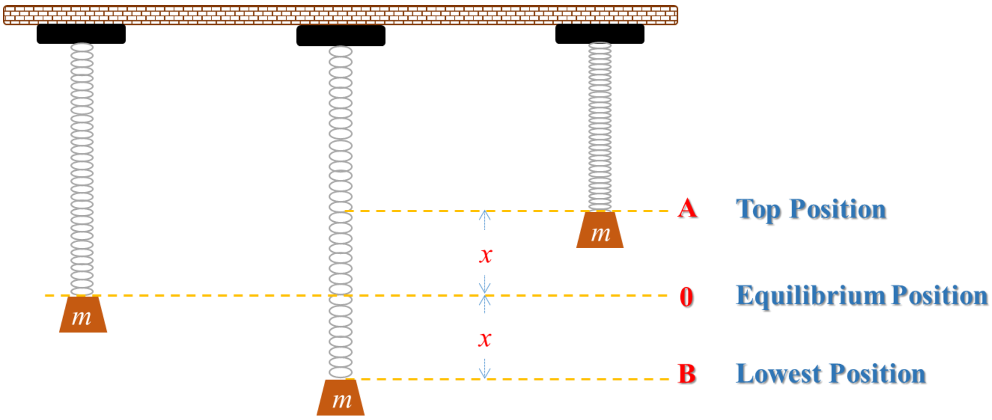


Figure 6.3. Simple harmonic motion of the body of mass *m* between the points *A* and *B*.

The time for one complete cycle of a body in a simple harmonic motion is called “***period, T***” and the number of cycles per unit time is called “***frequency, f***”. The relationship between period and frequency is given by

$$T = \frac{1}{f} \cdot \tag{6.5}$$

The period is determined by the spring constant and the total mass connected to the spring:

$$T = 2\pi\sqrt{\frac{m}{k}} \cdot \tag{6.6}$$

Solving Eq. 6.6 for the mass of the body gives;

$$m = \frac{k}{4\pi^2}T^2. \tag{6.7}$$

According to Eq. 6.7, if $m = f\left(T^2\right)$ graph is plotted, one obtains a straight line and the spring constant *k* can be found experimentally from the slope of the graph;

$$m = \tan\alpha = \frac{k}{4\pi^2} \cdot \tag{6.8}$$

6.1.2. Springs Connected in Series and Parallel

As seen in Figure 6.4, a new mechanical spring system that has a different spring constant k can be created by connecting the springs in serial and parallel. Two springs with spring constants k_1 and k_2 connected to end-to-end is called "***series connection***"; the system formed by connecting the ends of the side-by-side is called "***parallel connection***".

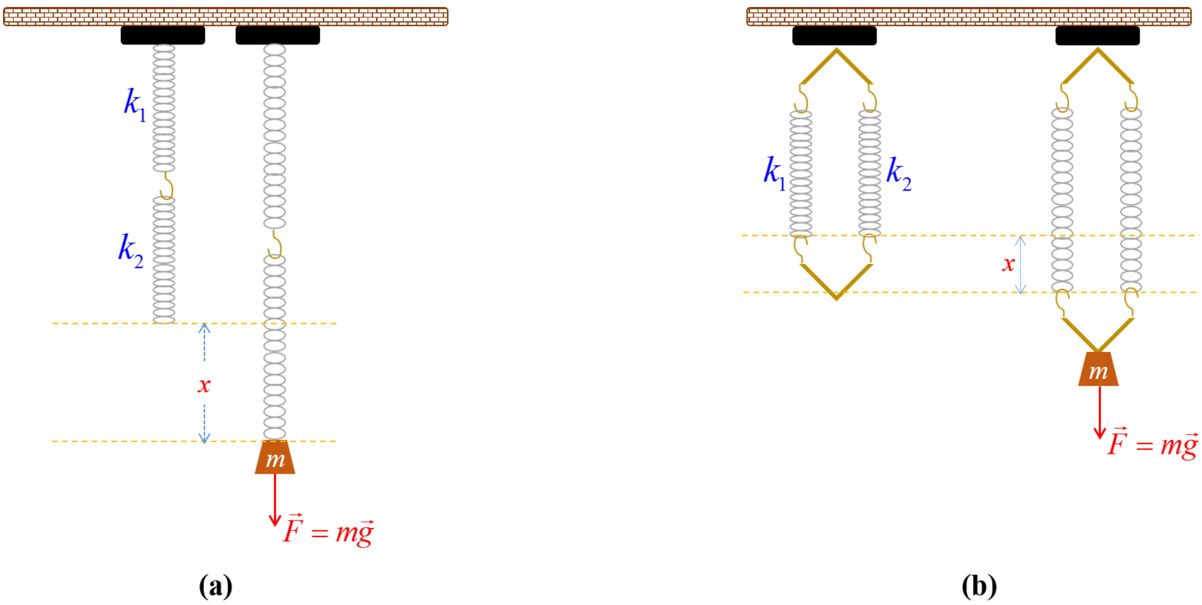


Figure 6.4. (*a*) Springs connected in series, **(*b*)** Springs connected in parallel.

As seen in Figure 6.4 (a), in series connection, the same force acts on each spring and the total elongation (or compression) of the spring system depends on the amount of elongation (or compression) of each spring. In this case, the mass m moves an amount of $x = x_1 + x_2$ downward.

$$F = F_1 = F_2. \tag{6.9}$$

$$x = x_1 + x_2. \tag{6.10}$$

According to Eq. 6.1, one can writte;

$$x = \frac{F}{k}; \qquad x_1 = \frac{F_1}{k_1}; \qquad x_2 = \frac{F_2}{k_2}. \tag{6.11}$$

Plugging Eq. 6.11 into Eq. 6.10, the equivalent spring constant k of the spring system for series connection is obtained as;

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} \tag{6.12}$$

or

$$k = \frac{k_1 k_2}{k_1 + k_2}. \tag{6.13}$$

As seen in Figure 6.4 b, when two springs are connected in parallel, the forces acting on each spring is different (F_1 and F_2) and when the elongation of the system is x, the elongation of each spring becomes x:

$$F = F_1 + F_2 \tag{6.14}$$

$$x = x_1 = x_2. \tag{6.15}$$

According to Eq. 6.1, one can writte;

$$F = kx; \qquad F_1 = k_1 x; \qquad F_2 = k_2 x. \tag{6.16}$$

Plugging Eq. 6.16 into Eq. 6.14, the equivalent spring constant k of the spring system for parallel connection is obtained as;

$$k = k_1 + k_2. \tag{6.17}$$

According to E.q. 6.17, when the springs are connected in parallel, the equivalent spring constant k will be greater than spring constant of each spring (k_1 and k_2).

6.2. MOVABLE PULLEYS

Pulleys which can both roll and rise or fall when a string passes around them are called "***movable pulleys***". A mass connected to a movable pulley rises with the pulley (Figure 6.5). If the weight of the pulley is neglected, the force acting on the string is equal to the half of the weight of the mass:

$$F = \frac{W}{2}, \tag{6.18}$$

where F is the applied force and W is the weight of the mass.

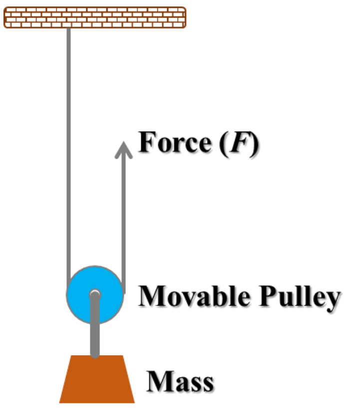


Figure 6.5. A hanged mass moving together with the pulley.

In a movable pulley system, the ratio of weight to the applied force is known as "***gain of force***":

$$\text{Gain of Force} = \frac{W}{F}. \tag{6.19}$$

In movable pulleys, the amount of the pulling distance where the force F is applied, is twice the amount of the distance that the mass moves:

$$\text{The amount of the pulling of string} = 2\left(\text{Path of the mass}\right) \tag{6.20}$$

According to Eq. 6.20, in order to lift a mass connected to a movable pulley up to a height h, the string on which the force is applied, must be pulled up to a height $2h$. Movable pulleys provide advantage of two in force, but disadvantages of two in path.

6.3. PULLEY BLOCKS

As seen in Figure 6.6, a system which consists of fixed and movable pulleys is called "***pulley blocks***". The string used in pulley block system passes all around the pulleys.

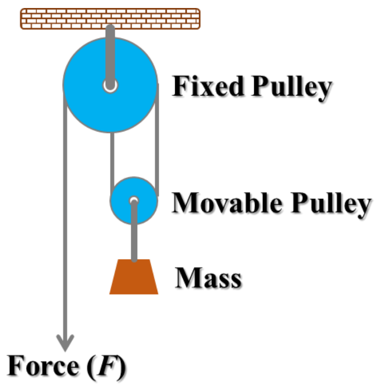


Figure 6.6. A pulley system consisting of two pulleys.

The gained force provided by the pulley blocks, depending on the number of string between fixed and movable pulleys is given by

$$F = \frac{W}{n} \tag{6.21}$$

where F is the applied force, W is weight of the mass, and n is the number of strings between fixed and movable pulleys. Eq. 6.21 is valid for the pulley blocks where pulley weights and friction are negligible. If pulley weights are not negligible, then the weights of the moving pulleys are added to the weight of the hanging mass.

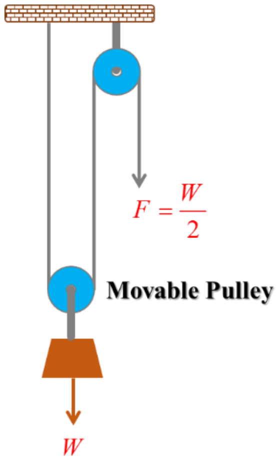


Figure 6.7. A movable pulley system held in equilibrium with a force $\vec{F}$.

In the pulley block system consisting of a fixed and movable pulleys as given in Figure 6.7, the balancing force (*F*) is equal to the half of the weight of the hanging mass, since the number of strings between the fixed and the movable pulleys is 2.

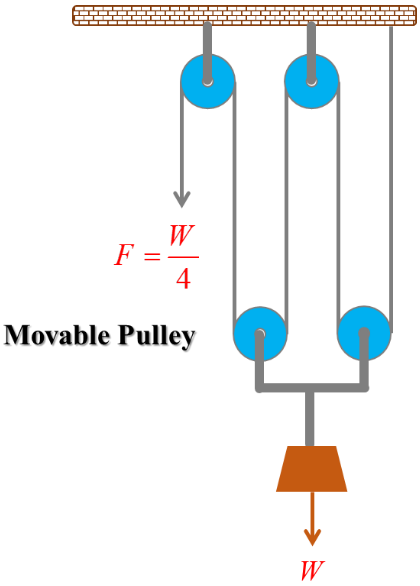


Figure 6.8. A pulley system consisting of two fixed and two movable pulleys.

In the pulley-block system given in Figure 6.8, the force holding the mass is the quarter of the weight of the mass, since the number of strings between the fixed and movable pulleys is 4. According to this, the path of the mass (*h*) is equal to quarter the amount of pulling distance of the string (*x*) on which the force is applied ($x = 4h$). As a result, in pulley block systems, there is an advantage in force but disadvantage in path.

PROCEDURE:

A. A Single Spring

A1. Hang the spring on a fixed point. As shown in Figure 6.9, set the bottom of the spring free as a reference point before hanging any mass.

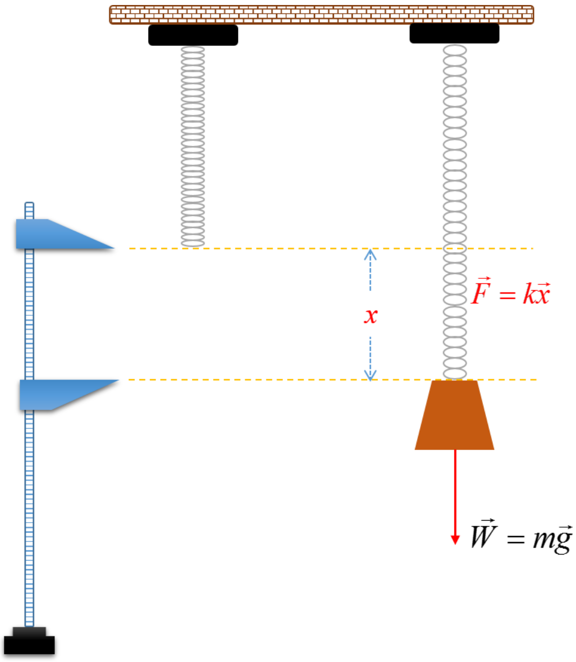


Figure 6.9. Experimental setup for determination of spring constant.

A2. Hang a mass of 200 g to the end of the spring. Wait until the mass-spring system is in equilibrium position, and then measure the elongation of the spring (x) and record it in Table 6.1.

A3. Repeat the same procedures with the masses 100 g and 50 g, and record the results in Table 6.1.

A4. Use Eq. 6.3 to calculate the force F applied on the spring by each mass (due to the weight of the masses), and record them in Table 6.1.

A5. Use the equilibrium condition Eq. 6.4 to calculate the spring constant for each hanging mass for 50, 100, and 200 g, and record them in Table 6.1. Compare your results.

A6. By using the data you obtained in Step 2 and 4, plot the graph $F = f(x)$. Calculate the spring constant k' by using the slope of the graph, and record it in Table 6.1.

A7. Compare the spring constants k and k', and make comments.

B. Spring System in Series

B1. Choose two springs with different spring constants. First, hang the first spring on a fixed point. Then, set the bottom of the spring free as a reference point before hanging any mass.

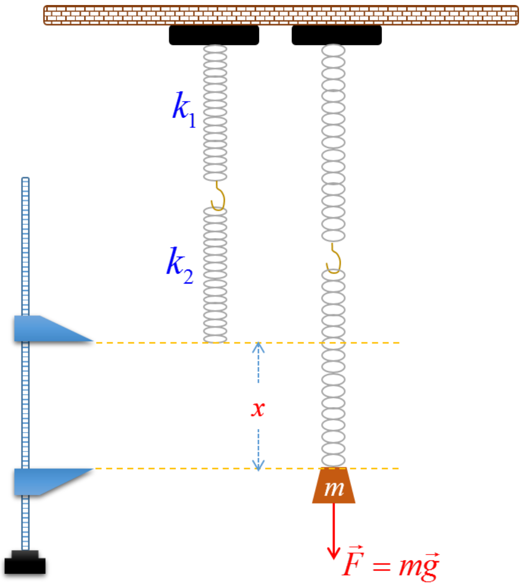


Figure 6.10. Experimental setup for the spring system of series connection.

B2. Hang a mass of 200 g to the end of the spring. Wait until the mass-spring system is in equilibrium position, and measure the elongation of the spring *x*. Then record it in Table 6.2.

B3. Repeat the steps above for the second spring and record the results in Table 6.2.

B4. Connect the first and second spring in series, as shown in Figure 6.10, set the bottom of the spring system free as a reference point before hanging any mass.

B5. Hang a mass of 200 g to the end of the spring. Wait until the mass-spring system is in equilibrium position, and measure the elongation of the spring *x*. Then record it in Table 6.2.

B6. Use Eq. 6.3 to calculate the force *F* applied on the first spring, second spring, and the series-connected spring system by the 200-g mass, and record them in Table 6.2.

B7. Use Eq. 6.4 to calculate the k_1 value for the first spring, k_2 for the second spring, and k for the series-connected spring system, and record them in Table 6.2.

B8. Use Eq. 6.12 to calculate the equivalent spring constant theoretically, and record it as k_{theory} in Table 6.2.

B9. Compare the theoretical and experimental results and make comments.

B10. Repeat the same procedure for masses 100 g ve 50 g, and record your results in Table 6.2.

C. Spring System in Parallel

C1. We calculated the values k_1 and k_2 for the first and second springs in series connected spring system before. Now connect the same springs in parallel as shown in Figure 6.11, set the bottom of the spring free as a reference point before hanging any mass.

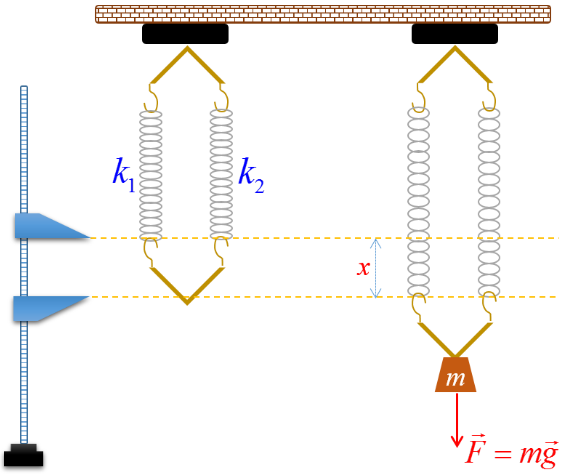


Figure 6.11. Experimental setup of parallel spring system.

C2. Hang a mass of 200 g to the end of the spring. Wait until the mass-spring system is in equilibrium position, and measure the elongation of the spring x. Then record it in Table 6.3.

C3. Use Eq. 6.3 to calculate the force F applied on the parallel-connected spring system by the 200 g mass, and record it in Table 6.3.

C4. Use Eq. 6.4 to calculate the spring constant k for the parallel-connected spring system, and record it in Table 6.3.

C5. Use Eq. 6.17 to calculate the equivalent spring constant theoretically for the parallel-connected spring system and record it as k_{theory} in Table 6.3.

C6. Compare the theoretical and experimental results and make comments.

C7. Repeat the same procedure for masses 100 g ve 50 g, and record your results in Table 6.3.

D. Determination of Oscillation Period

D1. As shown in Figure 6.3, hang a mass of 200 g to the end of the spring that you will use, and wait until the mass-spring system is in equilibrium position.

D2. Pull down the mass from the equilibrium position to a distance of 2 or 3 cm, and start the chronometer at the time when you release the mass.

D3. Measure the time elapsed t for "*10 full oscillations*" using a chronometer.

D4. Use $T = \dfrac{t}{10}$ to calculate the *period* T, and record it in Table 6.4.

D5. Find the value of T^2 and record it in Table 6.4.

D6. Use Eq. 6.6 to calculate the spring constant k for the spring that you use, and compare it with the value that you found using Hooke's Law before, and make comments.

D7. Repeat the same procedure for masses 100 g and 50 g, and record your results in Table 6.4.

D8. Plot the graph $m = f\left(T^2\right)$. Calculate the spring constant k' by using the slope of the graph, and record it in Table 6.4.

D9. Compare the spring constants k and k', and make comments.

E. Fixed Pulley

E1. Plug the fixed pulley to the assembly you will use, and pass a string all around the pulley.

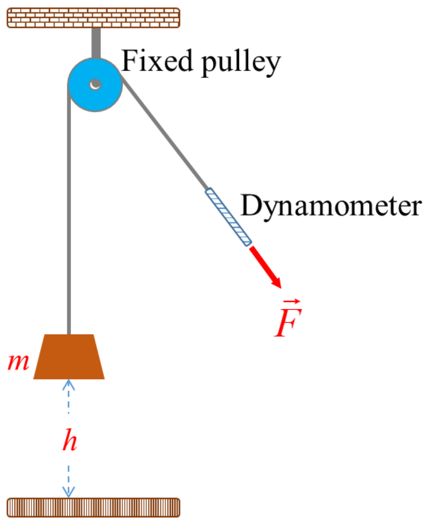


Figure 6.12. Experimental setup of a fixed pulley.

E2. Connect a mass 50 g to the end of the string, and the dynamometer to the other end. Keep the system in equilibrium by pulling the mass a little.

E3. Read the force F from the dynamometer while the system is in equilibrium, and record it in Table 6.5.

E4. Calculate the force of gravity acting on the mass using $W = mg$, and record it in Table 6.5.

E5. Rise the mass h = 10 cm from the ground by pulling the string that is connected to the dynamometer, measure the amount of pulling x on the string with a ruler corresponding to the path of the mass (h=*10* cm), record it in Table 6.5, and make comments on your results.

E6. Repeat the same procedure for masses 100 g and 200 g, and record your results in Table 6.5.

F. Pulley Blocks

F1. Install the pulley system as shown in Figure 6.13.

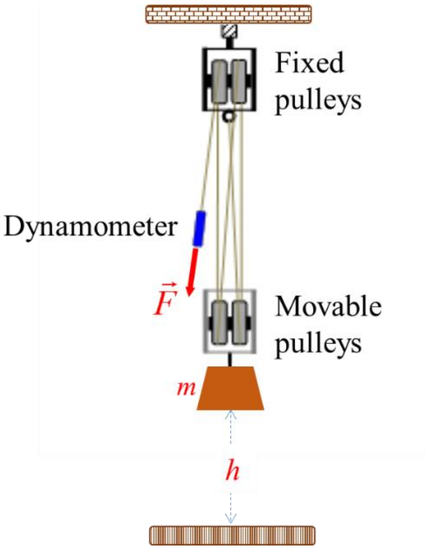


Figure 6.13. Experimental setup of pulley block that consists of two fixed and two movable pulleys.

F2. Connect a mass 50 g to one end of the string, and the dynamometer to the other end. Keep the system in equilibrium by pulling the mass a little.

F3. Read the force F from the dynamometer while the system is in equilibrium, and record it in Table 6.6.

F4. Calculate the force of gravity acting on the mass using $W = mg$, and record it in Table 6.6.

F5. Rise the mass $h = 10$ cm from the ground by pulling the string that is connected to the dynamometer, measure the amount of pulling x on the string with a ruler, corresponding to the path of the mass ($h = 10$ cm), record it in Table 6.6, and make comments on your results.

F6. Repeat the same procedure for masses 100 g and 200 g, and record your results in Table 6.6.

EXPERIMENTAL DATA:

TABLE 6.1

m (g)	x (cm)	F (N)	k (Nm^{-1})	k' (Nm^{-1})

TABLE 6.2

m (g)	First Spring			Second Spring			Series Connection			k_{theory} (Nm^{-1})
	x (cm)	F (N)	k_1 (Nm^{-1})	x (cm)	F (N)	k_2 (Nm^{-1})	x (cm)	F (N)	k_{eq} (Nm^{-1})	

TABLE 6.3
(Parallel Connection)

m (g)	x (cm)	F (N)	k_{eq} (Nm^{-1})	k_{theory} (Nm^{-1})

TABLE 6.4

m (g)	T (s)	T^2 (s^2)	k (Nm^{-1})	k' (Nm^{-1})

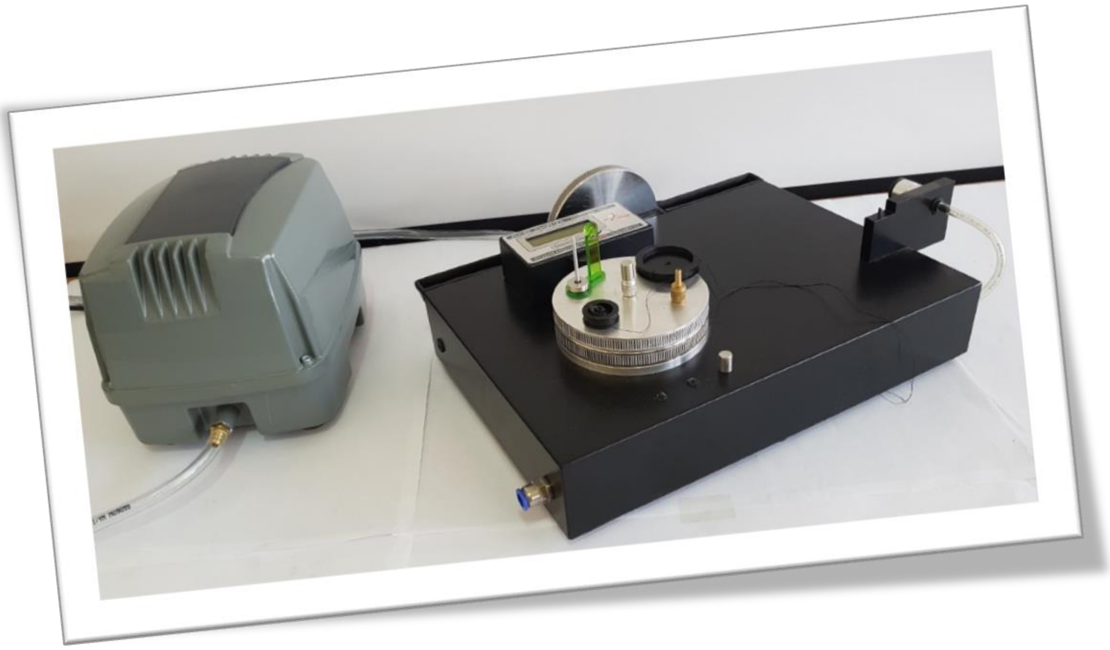
TABLE 6.5

m (g)	F (N)	W (N)	h (cm)	x (cm)
			10	
			10	
			10	

TABLE 6.6

m (g)	F (N)	W (N)	h (cm)	x (cm)
			10	
			10	
			10	

ROTATIONAL MOTION



M7

ROTATIONAL MOTION M7

PURPOSE OF THE EXPERIMENT:

1. To learn the concept of angular velocity and angular acceleration.

2. To investigate of conservation of angular momentum.

3. To determine the moment of inertia.

4. To study rotational kinetic energy and its conservation.

EQUIPMENT FOR THE EXPERIMENT:

The experiment kit of rotational motion (the kit will be introduced later)

THEORY:

7.1. UNIFORM CIRCULAR MOTION

Motion of an object with a constant linear velocity around a circle is called as "***uniform circular motion***". As shown in Figure 7.1, even though the magnitude of the velocity vector is constant during the motion, it changes with time because its direction changes. Therefore, the motion has a constant acceleration. In the uniform circular motion, the object has "***centripetal acceleration***, $\vec{a}_r$" and its magnitude is given by

$$a_r = \frac{V^2}{r} \tag{7.1}$$

where V is linear (tangential) velocity and r is the radius of the circular orbit. The direction of the centripetal acceleration is radially inward (towards to the center of the orbit).

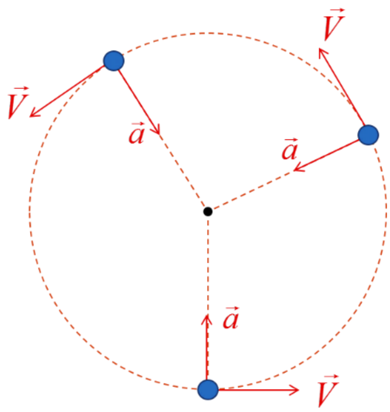


Figure 7.1. Uniform circular motion.

"Period, T" is defined as the time taken for one complete cycle by an object in a uniform circular motion and given by

$$T = \frac{2\pi r}{V}. \quad (7.2)$$

"Frequency, f" is defined as the number of cycles per unit time of an object in a uniform circular motion and is equal to the reciprocal of the period.

$$f = \frac{1}{T} \quad (7.3)$$

The SI units of the period and the frequency are second (s) and hertz (Hz), respectively ($1\ s^{-1} = 1$ Hz).

7.2. ANGULAR DISPLACEMENT, ANGULAR VELOCITY, AND ANGULAR ACCELERATION

An object that has a particular volume in space, a stable shape and is formed by numerous particles is called as "***rigid body***". Wheels, discs, and the World are some examples of rigid bodies. When a rigid body is rotating, each particle of the rigid body rotates with the same angle.

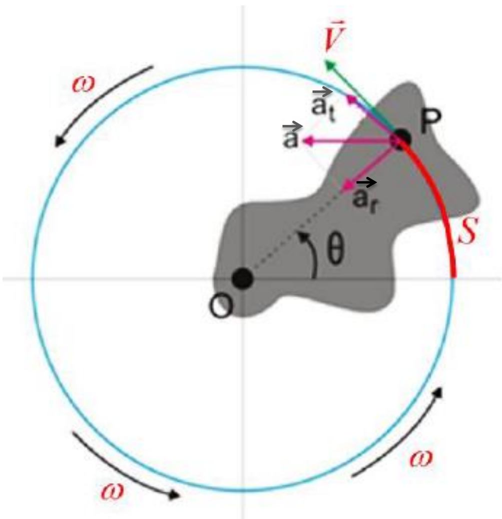


Figure 7.2. Angular dispalecement.

As seen in Figure 7.2, the distance covered by point P on a rigid body in a circular orbit of radius r is the arc length S. When the point P takes the distance S, its displacement θ is given by

$$\theta = \frac{S}{r}. \quad (7.4)$$

Here, θ is known as “angular displacement” in the unit of *radian* (rad).

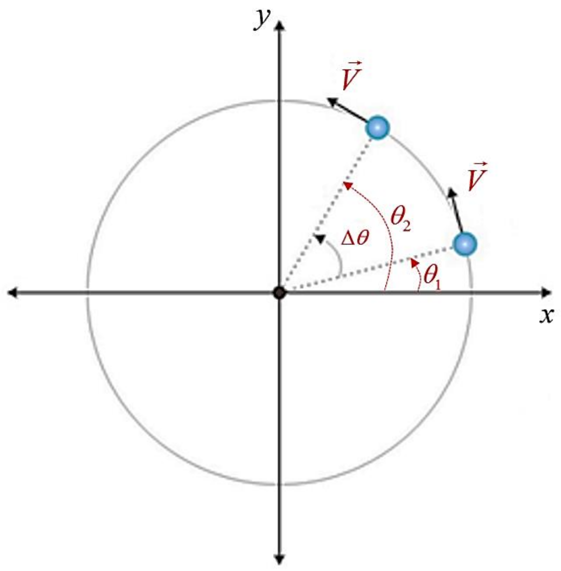


Figure 7.3. Angular displacement of point P at time t_1 and t_2.

As shown in Figure 7.3, if the angular displacement of point P at time t_1 is θ_1 and the angular dispalacement of point P at time t_2 is θ_2, *“average angular velocity”* is given by

$$\omega_{ort} = \frac{\Delta\theta}{\Delta t} = \frac{\theta_2 - \theta_1}{t_2 - t_1}. \tag{7.5}$$

“Instantaneous angular velocity” is the angular velocity at any time *t* and is given by

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}. \tag{7.6}$$

The SI unit of angular velocity is rads^{-1}.

If the angular velocity is not constant which means it changes with time, the point P will have a rotational motion with angular acceleration. **“*Average angular acceleration*”** is the change of the angular velocity ($\Delta\omega$) during the time period (Δt) and is given by

$$\alpha_{ort} = \frac{\Delta\omega}{\Delta t} = \frac{\omega_2 - \omega_1}{t_2 - t_1}, \tag{7.7}$$

where ω_1 is the angular velocity of point P at time t_1 and ω_2 is the angular velocity of point P at time t_2. ***“Instantaneous angular acceleration”*** is the angular acceleration at any time *t*. Instantaneous angular acceleration is obtained by taking the first derivative of the angular velocity or the second derivative of the angular displacement.

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta \omega}{\Delta t} = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} \cdot \quad (7.8)$$

The SI unit of angular acceleration is rads^{-2}. Each particle of the rigid body has the same angular velocity and angular acceleration. The relationship between the angular and the linear velocities of a point P on a rotating rigid body is given by

$$V = r\omega. \quad (7.9)$$

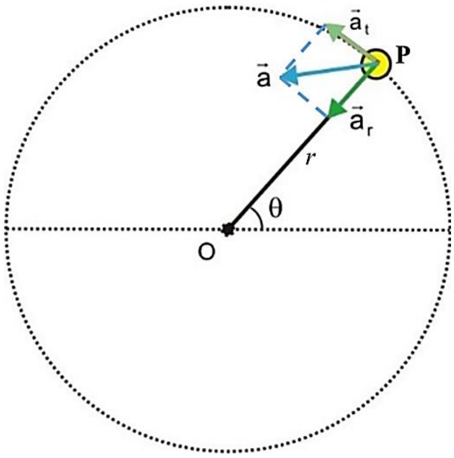


Figure 7.4. The tangential and the radial accelerations.

As shown in Figure 7.4, the acceleration $\vec{a}$ of a point P on the object which is undergoing uniform circular motion is composed of the angential ($\vec{a}_t$) and the radial ($\vec{a}_r$) accelerations. Magnitudes of the tangential (a_t) and the radial (a_r) accelerations are given by

$$a_t = r\alpha \quad (7.10)$$

and

$$a_r = \frac{V^2}{r} = r\omega^2, \quad (7.11)$$

respectively. While the direction of the tangential acceleration is always tangential to the orbit, the direction of the radial acceleration is always radially inward. Eqs. 7.9 - 7.11 provide the relationship between the rotational motion of a rigid body and the linear velocity of any point on the rigid body.

7.3. ROTATIONAL KINETIC ENERGY

A rigid body undergoing a rotational motion has a kinetic energy. Each particle of a rotating ridig body has different linear velocity, but they have the same angular velocity.

Therefore, the rotational kinetic energy is described in connection with the ***angular velocity***, but not with the linear velocity.

Assume that a rigid body consists of N particles. The rotational kinetic energy of the rigid body is the summation of the kinetic energy of each particle.

$$K = \frac{1}{2} m_1 V_1^2 + \frac{1}{2} m_2 V_2^2 + ... + \frac{1}{2} m_N V_N^2. \tag{7.12}$$

If Eq. 7.9 is plugged into Eq. 7.12, the rotational kinetic energy is given by

$$K = \frac{1}{2} m_1 \left(r_1\omega\right)^2 + \frac{1}{2} m_2 \left(r_2\omega\right)^2 + ... + \frac{1}{2} m_N \left(r_N\omega\right)^2 \tag{7.13}$$

$$K = \frac{1}{2}\left(\sum_{i=1}^{N} m_i r_i^2\right)\omega^2. \tag{7.14}$$

The expression inside the parenthesis in Eq. 7.14 is the multiplication of the mass of each particle which forms the rigid body and the square of the perpendicular distance of these particles to the axis of rotation. This term is known as ***"moment of inertia"*** and represented by *I*. The moment of inertia can be described by the resistance of a rigid body to the rotational motion. The moment of inertia of any rigid body depends on its shape, mass distribution, and the axes of rotation. The SI unit of moment of inertia is kg.m^2. The rotational kinetic energy of a rigid body in terms of moment of inertia is given by

$$K = \frac{1}{2} I\omega^2. \tag{7.15}$$

7.4. TORQUE

Torque is described as a phsical quantity which causes a rotation or the rotating effect of a force. Cross multiplication of an applied force on a rigid body and the perpendicular distance of the force from application point to the axis of rotation gives the torque:

$$\vec{\tau} = \vec{r} \times \vec{F} \tag{7.16}$$

As shown in Eq. 7.16, torque is a vector quantity, but each vector may not create a torque. To create a torque, the applied force must have a perpendicular distance to the axis of rotation. The SI unit of the torque is *N.m*.

Magnitude of the torque in terms of moment of inertia *I* and angular acceleration α is given by

$$\tau = I\alpha. \tag{7.17}$$

Eq. 7.17 is known as "***the equation of a rotatinal motion***". In a translational motion, Newton's second law is known as $F = ma.$ Equivalent equation of Newton's second law in a rotational motion is $\tau = I\alpha.$

7.5. ANGULAR MOMENTUM

Angular momentum in rotational motion is analogous to linear momentum in translational motion. Angular momentum vector $\vec{L}$ of a rigid body which has a mass of *m*, a position vector of $\vec{r}$ and a linear momentum of $\vec{P}$ is given by

$$\vec{L} = \vec{r} \times \vec{P}. \tag{7.18}$$

The SI unit of angular momentum is kgm^2s^{-1}.

The relationship between the torque $\vec{\tau}$ and the angular momentum $\vec{L}$ is given by

$$\vec{\tau} = \frac{d\vec{L}}{dt} \tag{7.19}$$

and Eq. 7.19 is an equivalent equation of Newton's Second Law in a rotational motion. The rate of change in the angular momentum of the rigid body or first derivative of the angular momentum with respect to time gives the torque. If the net torque due to net force on the rigid body is zero, the angular momentum is constant. This statement is known as "***the conservation law of angular momentum***". If $\vec{\tau} = \vec{0}$

$$\frac{d\vec{L}}{dt} = 0 \text{ and } \vec{L} = \text{constant. } (\textbf{\textit{Conservation of Angular Momentum}}) \tag{7.20}$$

Magnitude of the angular momentum of a rigid body in terms of moment of inertia *I* and the angular velocity ω is given by

$$L = I\omega. \tag{7.21}$$

If the multiplication of moment of inertia and angular velocity is constant, the angular momentum will be conservative. For example, when the moment of inertia of the rigid body increases, the angular velocity will increase for the angular momentum to be conservative.

PROCEDURE:

> **Clean both surfaces of all discs that will be used in order to remove dust and dirt particles. Otherwise, dirty disc surfaces will cause errors.**

A. Introduction of the Experimental Kit

Figure 7.5. The experimental kit of the rotational motion.

Figure 7.6. Discs and stoppers.

Figure 7.7. Masses and torque pulleys.

The materials numbered in Figure 7.5 are listed below:

1. A metal stopper that controls the bottom disc motion (long stopper). In Figure 7.5, if the metal stopper is put on the hole number 5, the bottom disc will rotate. Otherwise, it will stay fixed.

2. A metal stopper that controls the upper disc. This stopper supplies either the rotation of only upper disc or the rotation of both upper and bottom discs together. In order to have the rotation of upper disc independent of the bottom disc, a short stopper (number 3 in Figure 7.5) is put on to the upper disc and the stopper number 2 shown in Figure 7.5 is put on to this short stopper.
3. The stopper for the rotation of only upper disc. The upper disc rotates independent of the bottom disc when the stopper number 3 is put on to the upper disc.
4. Pulse counter displays the bar number passing in front of the sensor in 1 s. The screen shows each bar number during 2 s. If the button on the kit is up, the secreen will show the bar number passing in front of the upper sensor in 1 s. If the button on the kit is down, the secreen will show the bar number passing in front of the bottom sensor in 1 s. Sensors face to the disc number 9.
5. The hole on which the stopper number 1 is put.
6. The hole on which the stopper number 2 is put not to get lost.
7. The hole on which the stopper number 3 in Figure 7.5 and the stopper of number 6 in Figure 7.6 are put not to get lost.
8. The part where the compressor hose is connected. The plastic hose is attached by pushing inside. To take the hose out, the hose is pulled outside by pressing the blue part.
9. Fixed disc surface on which the bottom disc rotates.
10. The part that has small holes on and conveys the air coming from the compressor to both the upper disc and the bottom disc.
11. The pulley that is engraved for the string to pass through. When the torque pulleys are placed on the system, the string that has the mass on one end will pass through on it. The friction will be minimum due to air coming from the compressor.

Discs and stoppers shown in Figure 7.6 are listed below:

1. The bottom disc which is made of stainless steel, has 200 bars on its sides, a radius of 6.3 cm and a mass of 1.35 kg. The disc is placed onto the number 9 disc surface shown in Figure 7.5. The surface of the bottom disc shown in Figure 7.6 will be directed upward.
2. The upper disc which is placed onto the bottom disc. The upper disc has the same properties of the bottom disc. The upper disc is placed onto the bottom disc such that its surface shown in Figure 7.6 will be directed downward. The hole of smaller radius has screw steps. Hence, the screwed stoppers can be used. These stoppers cannot move due to air pressure. When

the stoppers are put on, be careful not to make it so tight because tight stoppers will ventilate the disc.

3. The upper disc that is made of aluminum, has 200 bars on its sides, having a small radius with screw steps, a radius of 6.3 cm and a mass of 470 g.

4. The stopper which allows to open and close the stopper number 6 which has a hole on the axis.

5. The metal stopper that controls the bottom disc rotation. If the stopper is put on to the hole number 5 in Figure 7.5, the bottom disc will rotate. Otherwise, the bottom disc will be stable.

6. The screw which has screw steps on the surface and a hole in its axis. The screw can be plugged into the discs numbers 1, 2, and 3. The screw provides the rotation of either the upper disc and the bottom disc together or only the upper disc independently of the bottom disc. If the stopper number 4 is put on the hole on the center of the disc, only the upper disc will rotate independently of the bottom disc. If the hole is open, both the upper disc and the bottom disc will rotate together.

7. The screw that is similar to the screw number 6, but has no hole in its axis. It will be used only if both discs will rotate independently.

Masses and torque pulleys shown in Figure 7.7 are listed below:

1. The torque pulley that is placed onto the upper disc. The face of the torque pulley shown in Figure 7.7 will be directed up. The torque pulley will be put on to the upper disc by using the stopper number 6 shown in Figure 7.6. While the stopper is put on to the upper side of the pulley, the black hoop on one end of the string will be placed on to the bottom side of the pulley. The string must pass through the hole on the pulley of radius 2.5 cm.

2. The torque pulley which has the same properties with the number 1 torque pulley except their radii.

3. The mass of 5 g. It is hanged on the blue apparatus seen in Figure 7.7.

4. The mass of 10 g. It is hanged on the blue apparatus seen in Figure 7.7.

5. The mass of 20 g. It is hanged on the blue apparatus seen in Figure 7.7.

6. String with a blue apparatus mass of 5 g on its one end to hang the masses. There is a black hoop to be attached to the torque pulley on its other end. It can be attached by using the stopper number 6.

Discs can rotate in four different ways:

1. **The upper and the bottom discs rotate together:** The upper disc is placed onto the bottom disc. The stopper number 1 is put on to the hole number 5. The stopper number 6 seen in Figure 7.6 with a hole on its axis is put on to the upper disc.
2. **The bottom disc is fixed, the upper disc rotates:** The hole number 5 in Figure 7.5 is kept open. The stopper number 4 is put on to the stopper number 6 and both of them are put on to the upper disc. Or the same procedure can be done by using only the stopper number 7.
3. **Both discs rotate independently:** The hole number 5 in Figure 7.5 is closed. The stopper number 4 is plugged into the stopper number 6 and both of them are put on to the upper disc. Or the same procedure can be completed by using only the stopper number 7.
4. **Both discs are fixed:** The hole number 5 in Figure 7.5 is kept open and the stopper number 6 in Figure 7.6 with a hole is put on to the upper disc.

B. Measurement of Angular Velocity

B1. Initially, remove the long stopper and put the short stopper on the white screw. Hence, bottom disc will be fixed and the upper disc will be free to rotate.

B2. Connect the compressor and turn it on.

B3. Hold on the upper disc with your hand stationary. Wait until you see the ‘zero’ value on the digital screen.

B4. Then, rotate the upper disc by applying a force with your hand.

B5. Read changing bar numbers β and record it in Table 7.1. Repeat the same steps 3 times with different applied force to rotate the upper disc.

B6. The values you read are the bar numbers passing in front of the sensor in 1 s. Find the angular speed of the disc by using this information and the following formula; $\omega = \left(\pi / 100 \right) \times \beta$ and record the results in Table 7.1.

$$\text{Angle Value}\left(^\circ\right) = \frac{360}{200}\beta$$

$$\text{Angle Value }\left(rad\right) = \text{Angle Value }\left(^\circ\right) \times \frac{2\pi}{360}$$

The angle value in radian per second gives the angular velocity:

$$\omega = \frac{\text{Angle Value }\left(rad\right)}{1\ s} = \frac{\pi}{100} \times \beta$$

C. Measurement of Rotational Moment of Inertia

C1. Setup the system such that the bottom disc will be fixed and the upper dics (aluminum) will be free to rotate.

C2. Assemble the torque pulley of small radius and the black plastic hoop on one end of the string which has a mass holder on the other end on the upper disc. Pass through the string from the hole on the torque pulley.

C3. Release the string after it passes through the engraved part of the pulley number 11 in Figure 7.5.

C4. Hang 10 g mass on the mass holder.

C5. Connect the compressor and turn it on.

C6. Rotate the disc clockwise. The string will be wound on the torque pulley. Let the mass holder rise up to the level of the pulley number 11.

C7. Hold on the upper disc until the digital screen shows 'zero' value.

C8. Release the system after you see 'zero' value on the screen.

C9. Read at least 3 values of bar numbers until the mass holder goes down and record the values in Table 7.2.

C10. Calculate the angular velocities from the values you read and by using $\omega = \left(\pi/100\right) \times \beta$. Record the values in Table 7.2.

C11. Plot the graph of $\omega = f\left(t\right)$.

C12. Find the angular acceleration (α) from the slope of the graph.

C13. Calculate the torque using Eq. 7.16 and record the result in Table 7.3. In this equation, the force F is the weight of the mass hung on, perpendicular distance r is the radius of torque pulley. The angle between the two vectors is 90^o.

C14. Find the rotational moment of inertia by using calculated torque and angular velocity found from the slope of the graph in Eq. 7.17. Record your result in Table 7.3 as $I_{experimental}$.

C15. Calculate theoretically the moment of inertia of the disc by using

$$I_{theory} = \frac{1}{2} M R^2$$

where M and R are the mass and the radius of the upper disc, respectively, and record the result in Table 7.3.

C16. Compare the theoretical and experimental moment of inertia values.

C17. Repeat the same steps with the torque pulley which has a bigger radius.

C18. Repeat the experiment with the stainless steel disc.

D. Conservation of Angular Momentum

D1. Place two stainless steel discs such that they will rotate independently. Shorter stopper and white screw will be used on the upper disc. Longer stopper will be put on to where it belongs.

D2. Rotate the upper disc, but be carefull for the bottom disc to stay stable.

D3. Read and record the bar number (β_i) in Table 7.4 when the stopper is closed. Calculate the angular speed (ω_i) of the upper disc by using the data in Table 7.4 with equation $\omega = \left(\pi/100\right) \times \beta$, and record the result in Table 7.4.

D4. Pull out the shorter stopper on the white screw while the upper disc is rotating. Thus, both discs will rotate together. When the bar numbers are equal, record the first equivalent bar number (β_s) in Table 7.4. Use this value in equation $\omega = \left(\pi/100\right) \times \beta$ to calculate the angular velocity of the upper disc (ω_s) and record it in the Table 7.4.

D5. Calculate the inital and the final angular momenta (L_i and L_s) by using Eq. 7.21. Record the results in Table 7.4 and make a discussion on it. When you caculate the initial momentum L_i of the upper disc, '*M*' is the mass of only the upper disc in moment of inertia formula. On the other hand, when you calculate the final momentum L_s, '*M*' is the total mass of the upper and the bottom discs because they rotate together.

D6. Repeat the same procedure with aluminum upper disc.

E. Calculation of Kinetic and Potential Energies

E1. Place aluminum discs such that the bottom disc is fixed and the upper disc is free to rotate.

E2. Assemble the torque pulley of small radius and the black plastic hoop on one end of the string with a mass holder on its other end on the upper disc. Pass the string through the hole on the torque pulley.

E3. Hang 10 g mass on the mass holder.

E4. Release the string after it passes through the engraved part of the pulley number 11 in Figure 7.5.

E5. Wound the string on the torque pulley by rotating the disc clockwise.

E6. Wait until you see 'zero' value on the secreen, then release the upper disc and record the first read bar number in Table 7.5.

E7. Calculate the angular velocity ω_0 from the values you read and using $\omega = \left(\pi/100\right) \times \beta$.

Record the values in Table 7.5.

E8. Calculate the angular acceleration, α, from the equation:

$$\tau = I\alpha = mgr$$

where I is the calculated moment of inertia for stainless steel and aluminum upper discs in the part C of the experiment.

E9. Calculate angular velocity ω for a given time t by using $\omega = \omega_0 + \alpha t$ and record the result in Table 7.5.

E10. Calculate the linear velocity using

$$V = \omega r$$

where r is the radius of the torque pulley and record the result in Table 7.5.

E11. Find the initial potential energy of the mass using

$$U = mgh = mh\left(\frac{\beta}{200} 2\pi r\right)$$

and record the result in Table 7.5.

E12. Find the kinetic energy of the mass after 1 s using

$$K = \frac{1}{2} mV^2 + \frac{1}{2} I\omega^2$$

and record the result in Table 7.5.

E13. Compare the potential and the kinetic energies and make a comment.

E14. Repeat the same experiment with a torque pulley of larger radius.

E15. Repeat the experiment with stainless steel upper disc. Use torque pulleys with large and small radius.

EXPERIMENTAL DATA:

TABLE 7.1	
Bar Number (β)	ω (rads^{-1})

TABLE 7.2								
t (s)	Bar Number (β)				ω (rads^{-1})			
	Aluminum Disc (r = 6.3 cm)		Stainless Steel Disc (r = 6.3 cm)		Aluminum Disc (r = 6.3 cm)		Stainless Steel Disc (r = 6.3 cm)	
	Torque Pulley (r=1.3cm)	Torque Pulley (r=2.5cm)	Torque Pulley (r=1.3cm)	Torque Pulley (r=2.5cm)	Torque Pulley (r=1.3cm)	Torque Pulley (r=2.5cm)	Torque Pulley (r=1.3cm)	Torque Pulley (r=2.5cm)
2								
4								
6								
8								
10								
$I_{theoretical}$					$I_{experimental}$			

TABLE 7.3				
Quantities	Aluminum Disc (r = 6.3 cm)		Stainless Steel Disc (r = 6.3 cm)	
	Torque Pulley (r = 1.3 cm)	Torque Pulley (r = 2.5 cm)	Torque Pulley (r = 1.3 cm)	Torque Pulley (r = 2.5 cm)
α (rads^{-2})				
τ (Nm)				
$I_{experimental}$ (kgm^2)				
$I_{theoretical}$ (kgm^2)				

TABLE 7.4						
Upper Disc	β_i	ω_i (rads^{-1})	L_i (kgm^{2}s^{-1})	β_s	ω_s (rads^{-1})	L_s (kgm^2 s^{-1})
Stainless Steel (r = 6.3 cm)						
Aluminum (r = 6.3 cm)						

TABLE 7.5							
Upper Disk	β	ω_0 (rads^{-1})	α (rads^{-2})	ω (rads^{-1})	V (ms^{-1})	U (J)	K (J)
Aluminum (r = 6.3 cm)							
Aluminum (r = 6.3 cm)							
Stainless Steel (r = 6.3 cm)							
Stainless Steel (r = 6.3cm)							

CENTRIPETAL FORCE

M8

CENTRIPETAL FORCE M8

THE PURPOSE OF THE EXPERIMENT:

To investigate centripetal force acting on an object in a circular motion and how it changes depending on speed, mass, and orbital radius of the object.

EQUIPMENT FOR THE EXPERIMENT:

Experimental setup of centripetal force

Mass set

Vernier-LabQuest-2 meter

THEORY:

8.1. ROTATIONAL MOTION

When an object of mass *m* moves with a constant linear speed ($\vec{V}$) in a circular orbit of the radius *r* as shown in Figure 8.1, the object has an acceleration ($\vec{a}_r$) directed towards the center of the circle during its motion. This acceleration is called as "***centripetal acceleration***" and its magnitude is given by

$$a_r = \frac{V^2}{r}, \tag{8.1}$$

where V is the linear velocity that is always tangent to the circular path of the object, and *r* is the distance from the object to the center of the circle.

According to the Newton's Second Law of Motion net force acting on a mass *m* in circular motion is defined as

$$F_r = m\frac{V^2}{r}\cdot \tag{8.2}$$

This force is called "***centripetal force***". The direction of the centripetal force is in the same direction as the centripetal acceleration that is towards the center of the circle. This indicates that an object making a circular motion is under the influence of a centripetal force. Centripetal force enforces an object to move in a circular orbit at a fixed radius. If the centripetal force acting on the object is removed, the object cannot naturally maintain its circular motion.

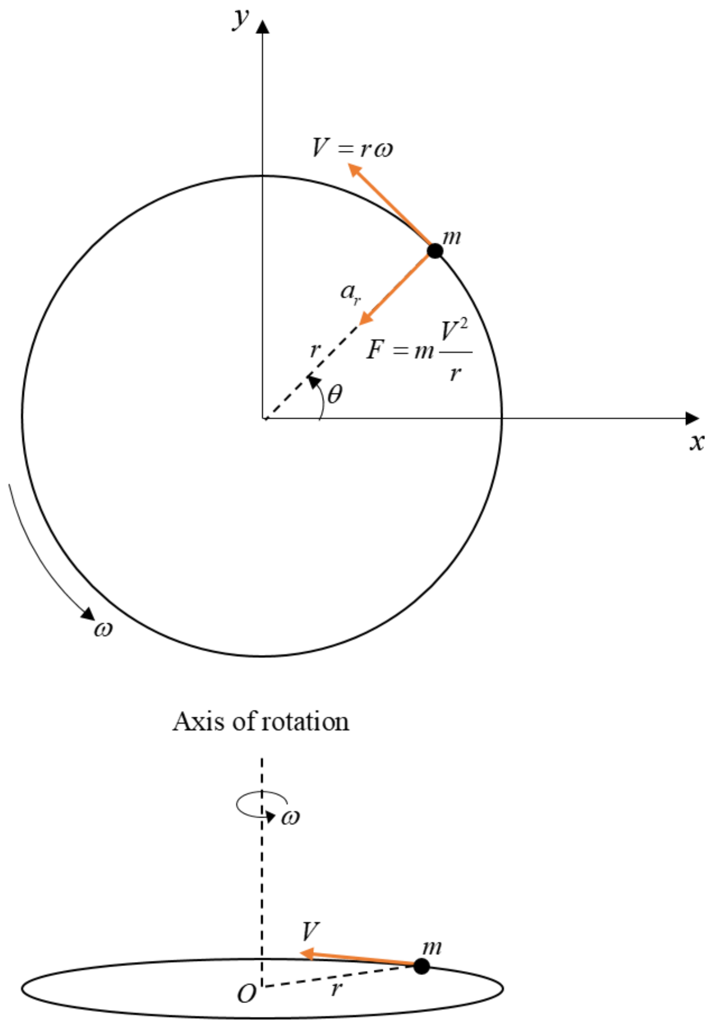


Figure 8.1. The magnitude and direction of centripetal force acting on an object in a circular motion.

If centripetal force acting on an object in Eq. 8.2 is reorganized, we obtain

$$\frac{F_r}{V^2} = \frac{m}{r}. \tag{8.3}$$

According to Eq. 8.3, if the ratio (m/r) is kept constant, the centripetal force (F_r) in circular motion changes with the square of the linear velocity (V^2) linearly, i.e., "a linear function". Therefore, the $F_r = f\left(V^2\right)$ graph is a straight line and its slope gives the ratio of the mass to the radius of the object in the circular motion (m/r).

If the magnitude of the linear velocity of mass m in a circular motion changes with time, the object has the tangential acceleration a_t as well as the centripetal acceleration a_r. The expressions that connect the tangential acceleration a_t and linear velocity V to the angular acceleration α and angular velocity ω are given by

$$a = r\alpha \tag{8.4}$$

$$V = r\omega. \tag{8.5}$$

When an object that is attached to one end of a rope with negligible mass rotates circularly, centripetal force acting on the object can be expressed in terms of the object's angular velocity, as well. If Eq. 8.5 is substituted into Eq. 8.2, the centripetal force becomes

$$F_r = m\frac{(r\omega)^2}{r} = mr\omega^2 = ma_r \cdot \tag{8.6}$$

According to this equation, the magnitude of centripetal acceleration is

$$a_r = r\omega^2. \tag{8.7}$$

For an object that is attached to one end of a rope with negligible mass in a circular motion, the centripetal force acting on the object is equal to the tension force (T) in the rope. The tension T is given by

$$T = F_r = m\frac{V^2}{r} = mr\omega^2 = ma_r. \tag{8.8}$$

Thus, the centripetal force acting on the object in circular motion depends on the mass, speed, and orbital radius of the object.

If an object starts its circular motion with an initial angular velocity ω_0 (or linear velocity V_0) at $t = 0$ and with a constant angular acceleration, its angular velocity ω at a given time t is defined as

$$\omega = \omega_0 + \alpha t. \tag{8.9}$$

If the initial angular velocity of the object is zero $(\omega_0 = 0)$, Eq. (8.9) becomes

$$\omega = \alpha t. \tag{8.10}$$

Similarly, the linear velocity V of the object is given by

$$V = at. \tag{8.11}$$

PROCEDURE:

1. As shown in Figure 8.2, connect the cables for the force measuring sensor and light gate to the "CH-1" and "DIG-1" ports of Vernier-labquest-2 device, respectively.

- Adjust the measuring range on the force sensor as *10* N.
- Connect the movable empty mass slot on the rotating platform to the force sensor over the frictionless pulleys with a rope of negligible mass.
- Adjust the distance between the movable empty mass slot and the first frictionless pulley by stretching the rope of negligible mass at most $r=10$ cm. Thus, you enable the empty mass slot to move in a circular motion of radius $r=10$ cm due to the tension force in the rope.
- Fix the location of the other empty mass slot on the rotating platform at $r=10$ cm away from the center. Make sure that this mass slot should not move on the rotating platform. Two empty mass slots on the platform are approximately m = 58 g.

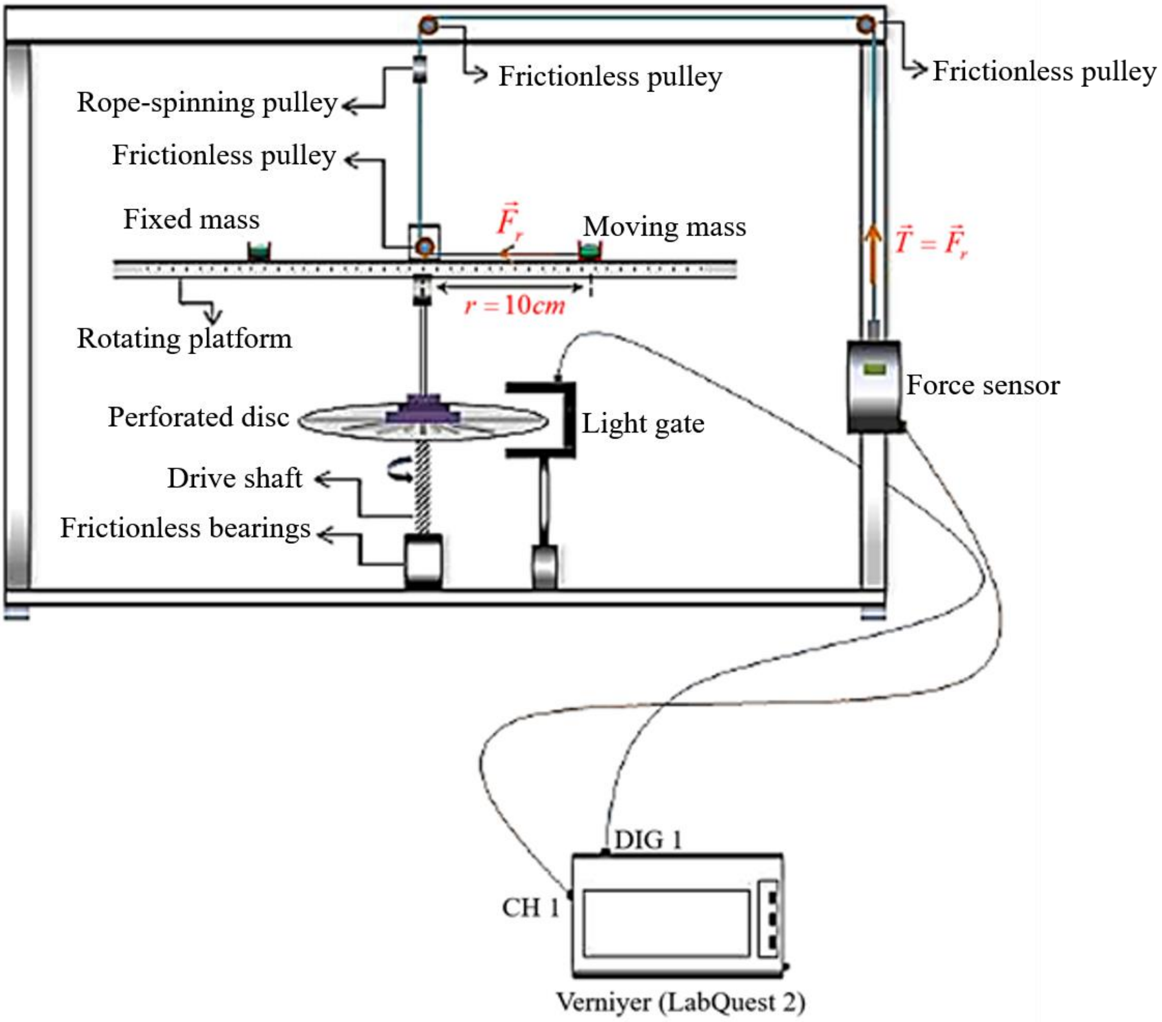


Figure 8.2. Experimental setup of centripetal force.

2. Place a $m' = 100$ g mass into both the moving mass slot and fixed mass slot.

- Sum the mass of the frictionless movable empty mass slot and the mass of the object used in the experiment ($m' = 100$ g).
- Record this total mass as total **test mass** *M* in Table 8.1.

3. Turn on the Vernier-LabQuest-2 meter by connecting the power cable.

The following measurement parameters are displayed on the opening screen of LabQuest-2:

CH-1: *Force*

DIG-1: *Gate State*

Mode: *Photogate Timing*

Timing: *Motion*

4. Introduce the orbital distance (the circumference) of the circular motion of radius *0.1* m to the LabQuest-2 device using the "**Mode**" button on the opening screen. For this operation, enter measurement codes as the following:

Photogate Mode: *"Motion"*

User defined: *"0.0628m"*

End data collection: *"with the stop button"*

and press the OK button.

NOTE: Because the radius of the circular motion is *r=10* cm, the linear path (the circle's circumference) of the total test mass in a full cycle becomes $x = 2\pi r = 0.628$ m. The total signal transition range of the light gate that is necessary for the circumference of the circular orbital (a full cycle) consists of 10 transition ranges on the perforated rotation disc. Therefore, the **linear path code** (*x*) corresponding to each light gate signal to be introduced to the LabQuest-2 device in a circular path of radius *r=10* cm becomes

$$x = 0.628/10 = 0.0628 \text{ m.}$$

5. Click on the "**Graph**" form in the upper right corner of the opening screen to display the graphic screen. Then, open the "Graph" menu and select the following steps:

Graph→Show Graph→Graph1

Graph h→Graph Options:

X-Axis column: "velocity"

"Autoscale from 0"

and press the OK button.

6. Enter the following option from the "**Data table (x/y)**" button in the Graph screen:

Table → *"**New Calculated Column**"*

Then, follow the following steps:

Name:V^2

Units: m^2/s^2

Displayed Precision: *3*

Decimal Places

Equation Type: Ax^2+Bx+C

$A=1$

$B=0$

$C=0$

Column for x: "*velocity*"

Thus, by using the linear velocity (V) at a given time t for the total test mass in a circular motion, LabQuest-2 software calculates the square of these velocities, i.e. V^2, in a new column named as "(V^2)".

7. Press again the "Show graph" – "Graph 1" button on the Graph menu.

- Click on the horizontal axis "x" of the graph and in the opened window, enter the predefined square of velocities:

  "$V^2\left(m^2/s^2\right)$"

- Similarly, click on the vertical axis "y" and in the opened window, enter the following option:

  "Force (N)".

  Thus, the axes for the graph of centripetal force as a function of velocity squared $\left(F = f\left(V^2\right)\right)$ are introduced to the LabQuest-2 device.

- Return to the opening screen of LabQuest-2 that shows the sections **Force** and **Gate State** by pressing the indicator icon in the upper left corner of the screen.

8. In the opening menu screen, click on the "**CH-1:Force**" section and in the opened window, do the reset operation "**ZERO**" prior to the measurement:

CH-1: Force→*ZERO*

- After the reset operation, start to rotate the platform by using a drive shaft and continue to rotate until there is a circular motion with a force between 6 – 8 N.
- Follow the force applied between 6 – 8 N in the LabQuest "**CH-1:Force**" section. Once the desired force value is obtained, release the system to have a free rotational motion.

- When the force falls to a value of $F = 5$ N in the LabQuest-2 screen, start to collect experimental data by clicking on the "**Start**" button in the lower left corner of the screen.
- When the force falls to a value of $F = 1$ N, stop the data collection process by clicking on the "**Stop**" button in the lower left corner of the LabQuest-2 screen.

9. Click on the following steps in the "**Analyze**" menu of the Graph screen:

Analyze →Curve Fit →Force.

Select the following steps in this submenu screen:

Fit Equation*: *Linear

With this command, the graph of force as a function of the velocity squared, its equation, and the slope of this curve appear on the screen.

- This slope is equal to the ratio of the total test mass M to the radius r of the circular motion.
- By using this slope, calculate the experimental test mass M' that undergoes a circular motion of radius r=*10* cm under a centripetal force and record it in Table 8.1.

10. Compare the values of known mass M and calculated test mass M' and make comments.

11. Repeat the experiment for masses $m' = 200$ g and $m' = 300$ g.

EXPERIMENTAL DATA:

TABLE 8.1.					
r (m)	m (kg)	m' (kg)	M (kg)	M' (kg)	ΔM (kg)

PHYSICAL PENDULUM AND MOMENT OF INERTIA

M9

PHYSICAL PENDULUM AND MOMENT OF INERTIA

M9

PURPOSE OF THE EXPERIMENT:

1. To find the moment of inertia of a circle by using a physical pendulum.
2. To examine the period and frequency of a physical pendulum.

EQUIPMENT FOR THE EXPERIMENT

Circle

Chronometer

THEORY:

9.1. PHYSICAL PENDULUM

An object hung by a point that does not pass through the center of mass (CM) will oscillate around a horizontal axis due to its mass. This system is called as "***physical pendulum***" or a "***composite pendulum***". When the object seen in Figure 9.1 is pulled from its equilibrium position and then is released, it will oscillate about an axis that passes through point A and is perpendicular to the page plane.

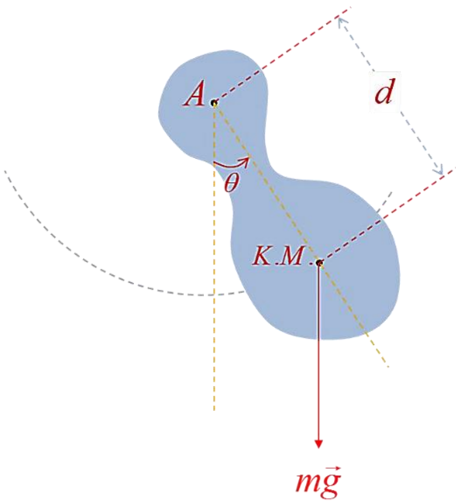


Figure 9.1. Physical pendulum.

The torque that makes the object oscillate is written as:

$$\vec{\tau} = \vec{r} \times \vec{F}. \tag{9.1}$$

The magnitude of this torque is given by

$$\tau = -mgd \sin\theta. \tag{9.2}$$

Here (-) sign appears because τ is a restoring force. For oscillations with small amplitudes, $(\theta < 5°)$, one may take $\sin\theta \sim \theta$. In this case, Eq. 9.2 becomes

$$\tau = -mgd\theta. \tag{9.3}$$

Given that I_A is the moment of inertia and α is the angular acceleration, the equation of motion for rotation is given by

$$\tau = I_A\alpha = I_A \frac{d^2\theta}{dt^2}. \tag{9.4}$$

Eq. 9.3 and Eq. 9.4 are equivalent. Thus this yields

$$-mgd\theta = I_A \frac{d^2\theta}{dt^2} \tag{9.5}$$

or

$$\frac{d^2\theta}{dt^2} + \frac{mgd}{I_A}\theta = 0. \tag{9.6}$$

The angular velocity of this oscillation is given by

$$\omega = \sqrt{\frac{mgd}{I_A}}. \tag{9.7}$$

Thus, the period of the physical pendulum for oscillations with small amplitudes $(\theta < 5°)$ can be written as:

$$T = 2\pi\sqrt{\frac{I_A}{mgd}}. \tag{9.8}$$

Here I_A is called "***moment of inertia***" about an axis passing through the point *A* on the object and perpendicular to the plane. In order to accelerate an object, we have to apply a force. In order to rotate an object with an angular acceleration, torque has to be applied about the axis passing through point *A* as shown in Figure 9.1. The applied torque is given in Eq. 9.4. The coefficient of proportionality for the torque-angular acceleration relation in Eq. 9.4 is called "***the moment of inertia of an object about a rotational axis***".

Angular acceleration is the change of the angular velocity with time. Moment of inertia may also be defined as a measure of resistance of an object against rotation, i.e. the changes that may occur in its angular velocity.

PROCEDURE:

1. Place the circle on the rod that is located at the edge of the table and remove it from its equilibrium position as to make a small angle with the vertical $(\theta < 5°)$, then release it to make an oscillation. Take care to keep the oscillation of the object within its own plane and not to make swaying motions.
2. Measure the time required for 10, 20, 30 and 40 oscillations with the help of a chronometer. Calculate the periods T_1, T_2, T_3, and T_4 for each oscillation and record them in Table 9.1. Calculate the average period T by using these four periods and record it in Table 9.1.
3. Measure the distance d from the point that the circle is suspended to its center and the mass of the circle (m) and record them in Table 9.1.
4. Substitute the values that you found in their respective positions into Eq. 9.8 and calculate the moment of inertia (I_A) about the point that the circle is suspended and record it in Table 9.1.

Error Calculation:

From Eq. 9.8,

$$I_A = \frac{T^2 mgd}{4\pi^2}$$

can be obtained. Then, the expression

$$\Delta I = 2T\Delta Tmgd + \Delta mT^2 gd + \Delta gT^2 md + \Delta dT^2 mg$$

is obtained. The relative error on the moment of inertia I_A becomes

$$\frac{\Delta I_A}{I_A} = 2\frac{\Delta T}{T} + \frac{\Delta m}{m} + \frac{\Delta g}{g} + \frac{\Delta d}{d} \cdot$$

Since the acceleration of gravity at the point where the experiment is conducted remains the same, $\Delta g = 0$ and the relative error is obtained as

$$\frac{\Delta I_A}{I_A} = 2\frac{\Delta T}{T} + \frac{\Delta m}{m} + \frac{\Delta d}{d} \cdot$$

ΔT standard deviation in the above relation is calculated from the period values found for four oscillations. However for Δm and Δd, the lowest indicator of the measuring device is taken.

EXPERIMENTAL DATA:

TABLE 9.1				
T_1 (s)	T_2 (s)	T_3 (s)	T_4 (s)	T (s)
d (m)	m (kg)	I_A		

TABLE 9.2			
ΔT	Δm	Δd	$\frac{\Delta I_A}{I_A}$

VISCOSITY

M10

VISCOSITY

M10

PURPOSE OF THE EXPERIMENT:

To find the viscosity coefficients of two different transparent liquids using the Stokes' Law.

EQUIPMENT FOR THE EXPERIMENT:

Three-chambered viscosity vessel

Two different transparent liquids

Balls

Ruler

Chronometer

Micrometer

Balance

THEORY:

When liquid layers are shifted against each other (for example when poured from a vessel to another vessel), forces against the movement arise between them. These forces are called internal friction or "***viscosity***". In other words, resistance of a liquid against the flow is called "***viscosity***".

As shown in Figure 10.1, let's consider two parallel plates, of equal surfaces, within a liquid. When a force $\vec{F}$ is applied to the upper plate it gains a $\vec{V}_1$ velocity, and liquid layers also gain gradually decreasing velocities starting from the top. This occurs due to the internal friction and at the end the bottom layer gains the smallest $\vec{V}_2$ velocity. Due to this distribution of the velocities, a liquid mass which has the shape of a cube when still takes the form of a parallelepiped prism when in motion. This type of flow is called "***laminar flow***". This flow also known as Newtonian flow or viscous flow. In this flow type, the net component of molecular velocities are in the direction of the flow, and the flowing liquid layers are not considered to crosscut each other. In cases in which the flow velocity of the liquid is not very high, only laminar type flow is observed.

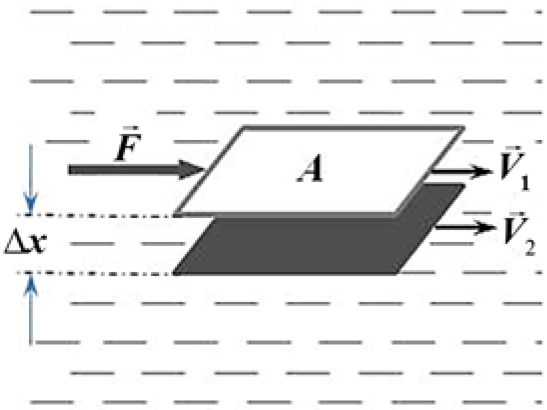


Figure 10.1. Liquid plates making laminar flow.

The velocity difference between two plates can be expressed as follows;

$$\Delta V = \frac{1}{\eta}\frac{F}{A}\Delta x \tag{10.1}$$

From this equation, we obtain:

$$F = \eta \frac{\Delta V}{\Delta x} A, \tag{10.2}$$

where $\frac{\Delta V}{\Delta x}$ is the velocity gradient; *A*, is the area of the plate, and η (eta) is the viscosity coefficient of the liquid.

Now let's obtain the viscosity coefficient of the liquid with the falling ball method. In Figure 10.2, the forces acting on a ball with a mass *m* thrown in a liquid are demonstrated.

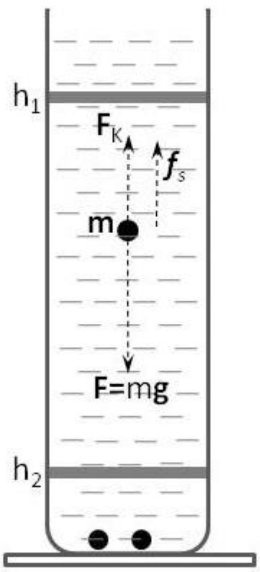


Figure 10.2. Forces acting on a ball with mass *m*.

When a ball with a radius of *r* is dropped in a liquid with a viscosity coefficient η, while the ball is falling with a constant velocity of $\vec{V}$, a frictional force

$$f_s = 6\pi r\eta V \tag{10.3}$$

acts on the opposite direction of the motion. This expression is known as ***"Stokes' Law"***. As it can be seen in Figure 10.2, the magnitude of ascending force F_k is obtained from the dynamic balance of the forces,

$$F = f_s + F_k \tag{10.4}$$

$$mg = 6\pi r\eta V + V_b\rho_s g \tag{10.5}$$

where ρ_s is the density of the liquid; $V_b = \frac{4}{3}\pi r^3$ is the volume of the ball and g is the acceleration of gravity. Given that the ρ is density of the ball, η is obtained as;

$$V_b\rho g = 6\pi r\eta V + V_b\rho_s g \tag{10.6}$$

$$V_b\left(\rho - \rho_s\right)g = 6\pi r\eta V \tag{10.7}$$

$$\frac{4}{3}\pi r^3\left(\rho - \rho_s\right)g = 6\pi r\eta V \tag{10.8}$$

$$9\eta V = 2r^2\left(\rho - \rho_s\right)g \tag{10.9}$$

$$\eta = \frac{2r^2\left(\rho - \rho_s\right)g}{9V}. \tag{10.10}$$

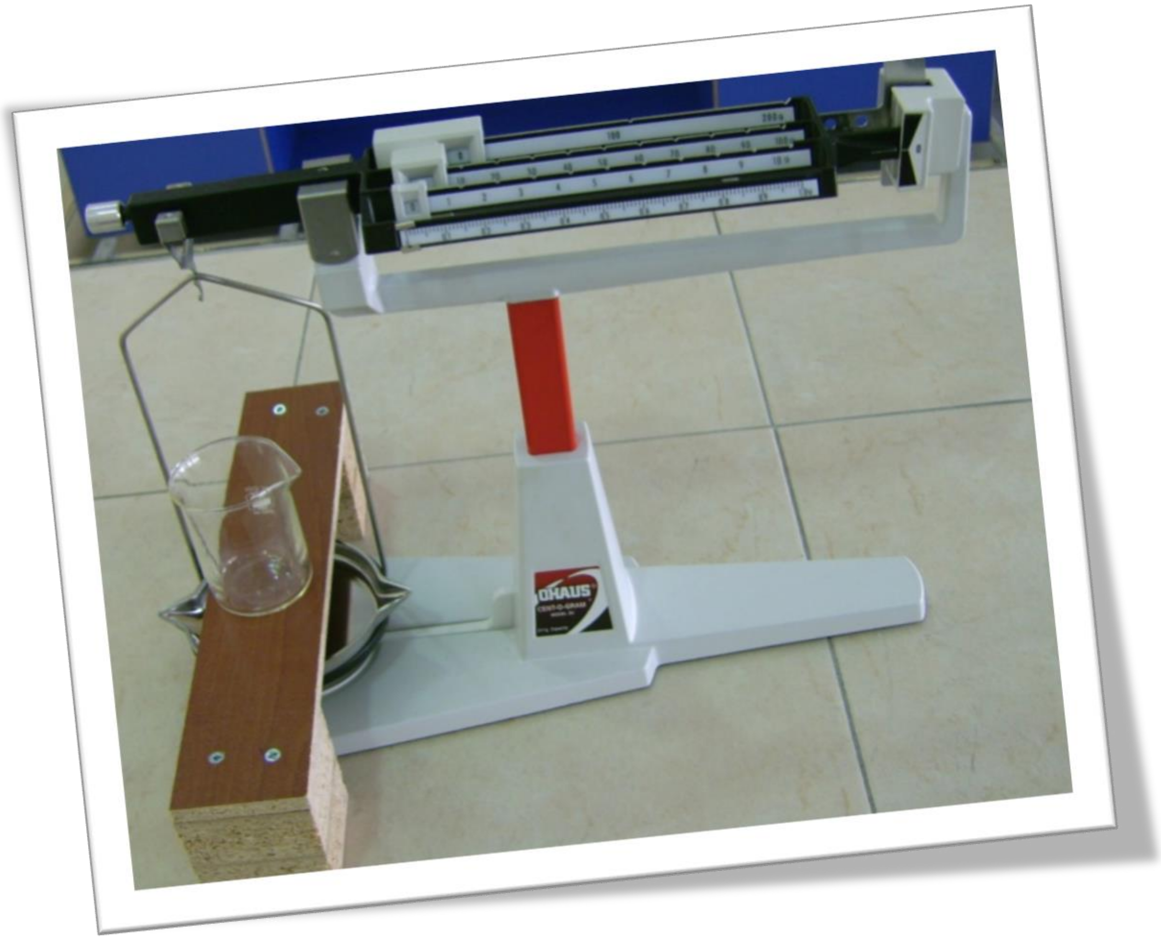
PROCEDURE:

1. Measure the diameters of 5 balls by using a micrometer. Take the average of the five measurements that you performed and calculate the average radius of a single ball from the expression $r = \frac{R}{2}$ and record it in Table 10.1.
2. Weigh all of the 5 balls on the precision balance and determine the total mass M. Calculate the average mass of a single ball $m = \frac{M}{5}$ and record it in Table 10.1.
3. By using the m and r values that you obtained for the ball, calculate the density ρ of the ball from $\rho = \frac{m}{\frac{4}{3}\pi r^3}$ and record it in Table 10.1.
4. Release one of the balls from rest for a free fall from a point near the surface and look through the horizontal plane that h_1 line in the viscosity vessel is present. Start the chronometer while the ball is passing through this plane and stop the chronometer when it passes through the h_2 line. Thus, the time t required for the ball to pass $s = h_1 - h_2$ distance between the marked lines is measured. Perform the same procedure for the other 4 balls and calculate average time t and record it in Table 10.1.
5. Calculate the velocity of the ball within the liquid using $V = \frac{s}{t}$ and record it in Table 10.1.
6. Learn the type of the liquid from the laboratory staff, determine the density ρ_s of the liquid by using Table 11.4, and calculate the viscosity coefficient of the liquid that you have been given by using Eq. 10.10 and record in Table 10.1.
7. Since viscosity coefficient changes with temperature, add the temperature of the laboratory (T) in your results, as well.
8. Perform the same experiment for the other liquid found in the viscosity vessel, compare the two results that you found and interpret your results.

EXPERIMENTAL DATA:

TABLE 10.1							
First Liquid				Second Liquid			
$T\left(°\mathrm{C}\right)$		$\rho_s\left(\mathrm{gcm^{-3}}\right)$		$T\left(°\mathrm{C}\right)$		$\rho_s\left(\mathrm{gcm^{-3}}\right)$	
$R\left(\mathrm{cm}\right)$		$s\left(\mathrm{m}\right)$		$R\left(\mathrm{cm}\right)$		$s\left(\mathrm{m}\right)$	
$r\left(\mathrm{cm}\right)$		$t\left(\mathrm{s}\right)$		$r\left(\mathrm{cm}\right)$		$t\left(\mathrm{s}\right)$	
$m\left(\mathrm{g}\right)$		$V\left(\mathrm{ms^{-1}}\right)$		$m\left(\mathrm{g}\right)$		$V\left(\mathrm{ms^{-1}}\right)$	
$\rho\left(\mathrm{gcm^{-3}}\right)$		η		$\rho\left(\mathrm{gcm^{-3}}\right)$		η	

ARCHIMEDES' PRINCIPLE AND DENSITY



M11

ARCHIMEDES' PRINCIPLE AND DENSITY

M11

PURPOSE OF THE EXPERIMENT:

1. To determine the densities of objects with different shapes and different types.

2. To determine the density of a liquid by using Archimedes' principle.

EQUIPMENT FOR THE EXPERIMENT

Centigram scale

Digital scale

Vernier caliper

Graduated cylinder

Overflow container

Beaker

Shaped object

Shapeless object

Distilled water

Liquid

THEORY:

An upward buoyant force acts on an object that is wholly or partially submerged into a fluid (liquid or gas). The buoyant force on the object is equal to the weight of fluid displaced by that object. Also, the volume of the displaced fluid equals the volume of the submerged part of that object. Therefore, if an object of volume V is completely submerged into a fluid of density ρ, buoyant force acting on the object is given by;

$$F = \rho V g. \tag{11.1}$$

If the weight of the object is greater than the buoyancy of fluid, the object sinks. In this case, the density of the object (ρ_{object}) is greater than that of the fluid (ρ_{fluid}). If the weight of object equals the buoyant force of the fluid, the object may stay in equilibrium everywhere inside the fluid. In this case, the density of the object is equal to the density of the fluid. Also, an object floats in the fluid as partially submerged if its density is less than that of the fluid. According to Eq. 11.1 buoyant force is given by

$$F = \rho V_{submerged}\, g \tag{11.2}$$

where $V_{submerged}$ is the volume of the submerged part of the object. The buoyant force occurs because the fluid exerts pressure on the entire submerged surfaces of the object. The pressure in a fluid increases with depth. Therefore, the pressure on the bottom surface of a submerged object is greater than that on the top surface since the bottom surface stays deeper in the fluid. The net force acting on the bottom and top surfaces is in upward direction and is equal to "***buoyant force of liquid***". Therefore, the resultant force due to the pressure acted on the object by the fluid gives the buoyant force. Since the buoyant force is always in the opposite direction to the force of gravity, it feels a noticeable reduction in the weight of an object immersed in a fluid compared to its actual weight as shown in Figure 11.1.

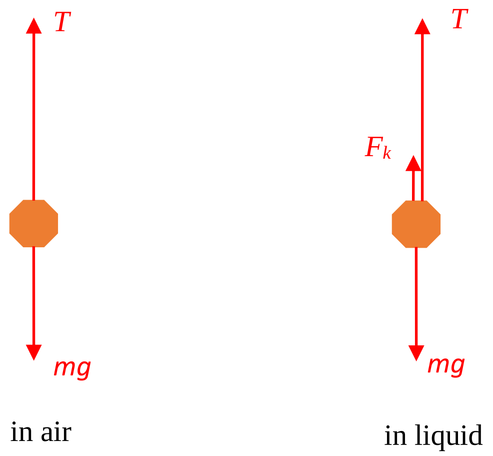


Figure 11.1. The forces acting on an object in air and in liquid.

By using weights of the object in air and in a liquid media, an expression for the density of the liquid can be derived as:

$$W_{air} - W_{liquid} = F_{buoyant} \tag{11.3}$$

$$\left(m_{object}\right)_{air}.g - m_{liquid}.g = \left(m_{object}\right)_{liquid}.g \tag{11.4}$$

$$\left(m_{object}\right)_{air} - m_{liquid} = \left(m_{object}\right)_{liquid} \tag{11.5}$$

$$\rho_{liquid} = m_{liquid} / V_{liquid} \quad \rightarrow \quad V_{liquid} = V_{submerged} \quad \rightarrow \quad m_{liquid} = \rho_{liquid}.V_{submerged}$$

$$\left(m_{object}\right)_{air} - \rho_{liquid}.V_{submerged} = \left(m_{object}\right)_{liquid} \tag{11.6}$$

$$\rho_{liquid} = \left[\left(m_{object}\right)_{air} - \left(m_{object}\right)_{liquid}\right] \Big/ V_{submerged} \tag{11.7}$$

PROCEDURE:

A. Determination of the Density of a Shapeless Object:

A1. Weigh the mass of the shapeless object (m_{object}) with a digital scale and record it on Table 11.1.

A2. Place the empty beaker under the pipe of the overflow container.

A3. Fill the overflow container slowly with distilled water just until the distilled water comes out of the overflow pipe.

A4. After you drain the accumulated water and dry the beaker, place again the empty beaker under the pipe of the overflow container.

A5. Release the shapeless object slowly into the overflow container.

A6. Measure the volume of the accumulated water (V_{water}) in the beaker with a graduated cylinder and record it in Table 11.1.

A7. Calculate the density of the object (ρ_{object}) and record it in Table 11.1.

B. Determination of the Fluid Density by Archimedes' Principle:

B1. Balance the empty centigram scale.

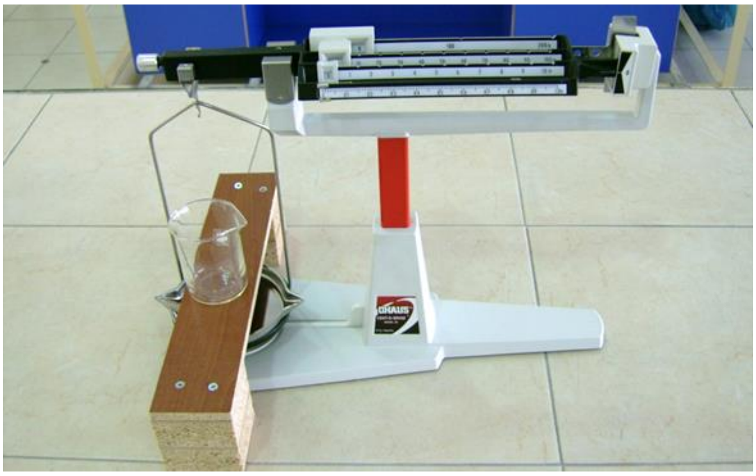


Figure 11.2. The experimental setup.

B2. Weigh the mass of the shaped object (m_{o_air}) by hanging it up on the centigram scale and record the weight in Table 11.2.

B3. Remove the shaped object from the scale.

B4. Place the wooden table as in Figure 11.2.

B5. Place the beaker filled with distilled water on the wooden table as shown in Figure 11.2.

B6. Submerge the shaped object totally into the distilled water and again hang it up on the centigram scale.

B7. Weigh the varied mass of the object (m_{o_water}) by centigram scale and record it in Table 11.2.

B8. Measure the outer (R) and inner (r) diameter and height (h) of the shaped object by Vernier caliper and record it in Table 11.2.

B9. Calculate the volume of the shaped object (V_{object}) by means of measured data and record it in Table 11.2.

B10. Calculate the density of water (ρ_{water}) and record it in Table 11.2.

B11. Dry the shaped object after removing it from the centigram scale.

B12. Repeat the steps from 3 to 12 for a different liquid and record your results in Table 11.3.

B13. Use Table 11.4 to identify the liquid.

EXPERIMENTAL DATA:

TABLE 11.1		
m_{object} (g)	V_{water} (cm^3)	ρ_{object} (gcm^{-3})

TABLE 11.2						
m_{o_air} (g)	m_{o_water} (g)	R (cm)	r (cm)	h (cm)	$V_{buoyant} = V_{object}$ (cm^3)	ρ (gcm^{-3})

TABLE 11.3							
m_{o_air} (g)	m_{o_water} (g)	R (cm)	r (cm)	h (cm)	$V_{buoyant} = V_{object}$ (cm^3)	ρ_{liquid} (gcm^{-3})	Which element or compound

TABLE 11.4. Density values for different objects.

SOLIDS (18°C) (gcm^{-3})

Aluminium	2.69	Potassium	0.53
Lead	11.35	Magnesium	1.74
Iron	7.5-7.8	Brass	8.1-8.6
Ice	0.92 (0°C)	Sodium	0.97
Glass	2.4-2.6	Nickel	8.85
Gold	19.29	Platinum	21.45
Wood (pine)	0.4-0.7	Silver	10.5
Copper	8.93	Zinc	7.12
Ebonite	1.2	Sugar	1.59
Diamond	3.01-3.52	Table salt	1.3-1.4

LIQUIDS (18°C) (gcm^{-3})

Ethylether	0.72	Glycerine	1.26
Ethylalcohol	0.79	Carbontetrachloride	1.59
Methylalcohol	0.79	Olive oil	0.91
Benzene	0.88	Mercury	13.55
Chloroform	1.49		

GASES (0°C and 760 mmHg) (kgm^{-3})

Ammonia	0.77	Carbondioxide	1.98
Argon	0.78	Air	1.29
Acetylene	1.17	Oxygen	1.43
Chlorine	3.22	Nitrogen	1.25
Helium	0.18	Hydrogen	0.09

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